

## FE studies on a coupled energetic-statistical size effect in concrete.

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A size effect in concrete elements causes that both the nominal structural strength  $\sigma_N$  and material brittleness decrease with increasing characteristic specimen dimension  $D$ . Thus, concrete becomes ductile on a small scale and perfectly brittle on a sufficiently large scale. The reasons of this behaviour are [1]: a) the presence of intense strain localization regions with a certain volume which precede macro-cracks and contribute to a deterministic size effect and b) a random distribution of material properties contributing to a statistical size effect. The first statistical theory introduced by Weibull [2] postulates that a structure is as strong as its weakest component. The statistical size effect by Weibull ignores a spatial correlation between local material properties and a deterministic size effect. Combining the energetic theory with a Weibull statistical theory, a general energetic-statistical theory was developed by Bazant [3].

Our extensive numerical FE investigations of a coupled energetic-statistical size effect in unnotched concrete beams of a similar geometry under quasi-static three point bending were performed within elasto-plasticity with non-local softening to obtain mesh-independent results [4], [5]. The FE analyses were carried out with four different beam sizes scaled in two dimensions. Deterministic calculations were performed with the uniform distribution of the tensile strength  $f_t$ . In turn, in stochastic calculations, the tensile strength  $f_t$  took the form of spatially correlated random fields described by a Gauss distribution (with the prescribed mean value  $\mu_{f_t}$ , standard deviation  $\sigma_{f_t}$  and correlation length  $l_{cor}$ ). In order to reduce the number of stochastic simulations, the stratified sampling technique was applied [4]. In calculations, the length of the spatial correlation  $l_{cor}$  ranged from 5 mm up to 150 mm and the material

variation coefficient  $cov = \sigma_{f_t} / \mu_{f_t}$  (keeping the constant  $\mu_{f_t}$ ) varied between 0.15 and 0.20. Next, FE calculations were carried out assuming simultaneously a cross-correlated spatial distribution [6] of the tensile strength  $f_t$ , fracture energy  $G_f$  and modulus of elasticity  $E$ . The effect of a strong and weak cross-correlation between  $f_t$ ,  $G_f$  and  $E$  was investigated.

The numerical outcomes showed a strong coupled energetic-statistical size effect in concrete beams [4]. The influence of the correlation length  $l_{cor}$  was found to be strong with reference to the mean nominal strength and its standard deviation. The higher  $l_{cor}$  the higher was the coefficient of variation. The increasing material coefficient of variation  $cov$  caused a reduction of the mean nominal strength. The higher material  $cov$  the higher was the standard deviation of resulting nominal strength. The simultaneously spatially varied parameters  $f_t$ ,  $G_f$  and  $E$  did not affect the mean stochastic nominal strength.

## References

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