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FORMULATION OF A NON-LINEAR SHELL FINITE ELEMENT ON THE LIE GROUP SE(3)

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The Lie group framework offers a number of advantages for the modelling and analysis of mechanical systems involving large rotations, e.g. for finite element models of systems with rigid bodies, kinematic joints, beams and shells. Firstly, the equations of motion are derived and solved directly on the nonlinear manifold, without an explicit parameterization of the motion, which leads to important simplifications in the formulations and algorithms [1]. Secondly, displacements and rotations increments in space and time can be systematically expressed in the material (body-attached) frame, which automatically filters the geometric non-linearities from the equations of motion.

The Special Euclidean group SE(3) is a Lie group which has been proven to be particularly convenient to describe spatial rigid body motions with rotation and position variables [2] and constraints in multibody systems [3]. Recently, the formulation of a beam finite element in this framework has been addressed [4]. The approach is based on an absolute coordinate finite element method, as in [5].

Under the shell assumption, the general study of a three dimensional continuum can be reduced to a two dimensional study on the so-called neutral surface of the shell and a one dimensional study over the thickness. The configuration of a point of the neutral surface of the shell is described by a mapping $\mathbb{R} \times \mathbb{R} \to SE(3) : (s,t) \to \mathbf{H}(s,t)$

$$\mathbf{H}(s,t) = \begin{bmatrix} \mathbf{R}(s,t) & \mathbf{x}(s,t) \\ \mathbf{0}_{3\times 1} & 1 \end{bmatrix} \tag{1}$$

where $\mathbf{x}(s,t)$ is the absolute coordinate of the point and $\mathbf{R}(s,t)$ is a rotation matrix which describes a local orientation with respect to the inertial axes. Thanks to a rigorous discussion of the polar decomposition of the deformation gradient, the local orientation is given a sound geometric interpretation. According to the Lie group structure, the derivative with respect to any parameter n can be expressed in the form of a left invariant vector field

$$\frac{\partial \mathbf{H}}{\partial n} = \mathbf{H}\tilde{\mathbf{n}} \tag{2}$$

where $\tilde{\mathbf{n}}$ is an element of the Lie algebra of $SE(3)$ which is naturally expressed in the local frame. The velocity and the deformation gradient can be introduced using Eq. (2) and are thus naturally expressed in the local frame. Due to the non-commutativity of the rotation group, complex Lie group features, such as the Lie bracket operator, are involved and must be treated adequately. Eventually, the equations of motion can be expressed in terms of the local velocities and local deformations only and are thus invariant under rigid body motions.

In order to discretize the resulting equations, consistent space and time discretization schemes must be adopted to deal properly with the Lie group structure. Extending the work achieved in [4] for a beam finite element formulation, a consistent discretization method for a shell element is introduced based on a space interpolation of nodal $SE(3)$ elements. In particular, this method handles naturally the non-commutativity and the invariance with respect to node numbering. It is shown that the local frame expression of the equations of motion is preserved. The geometric non-linearities in the equations of motion are thus reduced as well as the fluctuations of the iteration matrix during simulations, leading to a computationally efficient formulation. The proposed formulation is applied to standard examples in order to exhibit its consistency and its accuracy.

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