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MESHLESS FINITE DIFFERENCE METHOD – STATE OF THE ART

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MFDM [1, 3, 5, 16, 17] is the oldest and one of the most effective Meshless Methods [4, 6, 7]. Its current outlines, including the basic MFDM procedure, its various extensions and selected applications, demonstrating power, generality and versatility of the method competitive to main contemporary solution methods are briefly considered here. The MFDM may be applied in all formulations (strong, weak, mixed) dealing with derivatives. In particular the global/local MLPG [7] formulation proved to be especially effective.
The basic MFDM procedure consists of the following steps:

- Generation of nodal clouds
- Voronoi tessalation and Delaunay triangulation (in 2D)
- Selection of MFD stars
- Specification of degrees of freedom assumed
- MWLS or equivalent approximation
- MFD operators generation
- Integration (for global and global/local approaches)
- Generation of MFD equations (for both linear and/or nonlinear b.v.p)
- MFD discretization of boundary conditions
- Solution of discrete MFD equations and/or search for solution of a discrete optimization problem
- Postprocessing of the final results

Various MFDM extensions were developed, such as

- Higher Order MFD approximation including:
    - Multipoint Meshless FD approach [10, 11]
    - Use of correction terms [2, 9]
- MFD analysis of b.v. problems given in MLPG formulation [8]
- Error analysis and the adaptive approach [2, 9]
- Smoothing of experimentally measured data [12]
- Various MFDM and FEM combinations [13]

Presented are several examples selected out of the variety of MFDM applications. Beyond the typical structural analysis

- residual stresses in railroad rails and vehicle wheels [12, 14, 15]
- inflatable pneumatic structures [12], and
- experimental data smoothing [12]

are discussed here.

Altogether through years MFDM proved to be general, versatile, and effective solution method, competitive to the Boundary Element Method (BEM), other MMs and potentially also to FEM. Its further developments, and variety of applications are expected.

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