

THERMOELASTODYNAMIC CRACK ANALYSIS IN FUNCTIONALLY GRADED MATERIALS UNDER IMPACT LOADING

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In recent years, functionally graded materials (FGMs) received considerable research interests in materials and engineering sciences. FGMs represent a new class of high-performance composite materials formed by continuously variable composition of the constituents over volume [1]. In comparison to the conventional composite materials, FGMs possess many superior mechanical, thermal, corrosion-resistant and wear-resistant properties. FGMs can be widely applied in engineering structures and components such as electronic devices, blast protection, corrosion resistant coatings, wear-resistant coatings, thermal barrier coatings and biomaterials. As a representative example of FGMs, the ceramic/metal FGMs are compositionally graded from a ceramic phase to a metal phase. Ceramic/metal FGMs possess the desirable properties of metals such as high toughness, large mechanical strength and excellent bonding capability and high heat, wear and corrosion resistances of ceramics. An important application area of FGMs is their utilisation in innovative engineering structures and structural elements under severe mechanical and thermal impact loading conditions. Because of the inherent brittle nature of ceramics, cracks or crack-like defects may develop in the manufacturing phase or during their services. Therefore, the fracture and damage analyses of FGMs under extreme mechanical and thermal impact loadings are of particular importance to their thermal and mechanical integrity, functionality, reliability and durability in engineering applications. Such analyses may provide a fundamental understanding of the failure mechanisms of FGMs that is helpful in the design, optimization and innovative applications of FGMs.

The initial-boundary value problems of transient linear coupled thermoelasticity are described by a system of coupled partial differential equations with variable coefficients supplemented by prescribed initial and boundary conditions. Due to the high mathematical complexity of the corresponding dynamic thermoelastic problems for non-homogeneous FGMs, analytical methods can be obtained only for very simple geometry and loading conditions. In general cases, numerical and experimental methods have to be applied to fracture and fatigue analyses

in FGMs subjected to thermal and mechanical impact loadings.

In this paper, the two-dimensional transient linear coupled thermoelastic crack problem in continuously non-homogeneous, isotropic and linear elastic FGMs under a mechanical impact loading is investigated. The material properties of the FGMs are assumed to be continuous functions of the spatial coordinates, while Poisson's ratio is taken as constant. A boundary element method (BEM) is developed to analyze the responses of the crack with traction-free crack-faces. The transient linear coupled thermoelasticity is governed by the equations of motion and the thermal balance equation. The Laplace-transform technique is applied to eliminate the time-dependence in the governing equations. A boundary-domain integral representation is derived from the generalized Betti's reciprocal theorem by using the fundamental solutions for a homogeneous, isotropic and linear thermoelastic solid [2]. The boundary-domain integral equations (BDIEs) are obtained for mechanical and thermal field quantities [3-5]. Due to the material non-homogeneity, this approach leads to domain integrals involving the unknown quantities in addition to the conventional boundary integrals. The domain integrals are transformed into boundary integrals by using the radial integration method [6,7]. A collocation method is implemented for the spatial discretization of the boundary-domain integral equations. After the boundary-domain integral equations have been solved numerically in the Laplace-transformed domain, the final time-dependent solutions are obtained by applying the inverse algorithm of Stehfest [8]. A displacement extrapolation technique is used to compute the dynamic stress intensity factors. Numerical examples for the dynamic stress intensity factors are presented and discussed to demonstrate the accuracy and the efficiency of the present BEM. The influences of the material gradation and the mechanical impact loading on the dynamic stress intensity factors are investigated in details.

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