

WHY DO ENGINEERS MAKE ILL-CONDITIONED MODELS AND WHAT CAN WE DO ABOUT THEM?

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User errors in finite element models can arise arbitrarily while using FE software. These errors can lead to an ill-conditioned stiffness matrix during model analysis which, in turn, leads to inaccurate results. It is therefore necessary to ensure the stiffness matrix is well-conditioned.

In 2011, version 8.6 of the finite element-based structural analysis package Oasys GSA [3] was released. A new feature in this release was the estimation of the 1-norm condition number $\kappa_1(K) = \|K\|_1 \|K^{-1}\|_1$ of the stiffness matrix K of structural models by using a 1-norm estimation algorithm of Higham and Tisseur [1] to estimate $\|K^{-1}\|_1$. The condition estimate is reported as part of the information provided to engineers when they carry out linear/static analysis of models and a warning is raised if the condition number is found to be large. The inclusion of this feature prompted queries from users asking how the condition number impacted the analysis and, in cases where the software displayed an ill conditioning warning, how the ill conditioning could be “fixed”.

We describe a method that we have developed and implemented in the software that enables engineers to detect sources of ill conditioning in their models and rectify them. We give the theoretical background and illustrate our discussion with real-life examples of structural models to which this tool has been applied and found useful. Typically, condition numbers of stiffness matrices reduce from $O(10^{16})$ for erroneous models to $O(10^8)$ or less for the corrected model.

Our findings are published in [2].

REFERENCES

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