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A MULTILEVEL MONTE CARLO REDUCED BASIS METHOD FOR THE HDG APPROXIMATION OF STOCHASTIC ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

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Recently there has been a growing interest in quantifying the effects of random inputs in the solution of partial differential equations (PDEs) that arise in a number of areas, including fluid mechanics, elasticity, and wave theory to describe phenomena such as turbulence, random vibrations, flow through porous media, and wave propagation through random media. Monte Carlo based sampling methods, generalized polynomial chaos and stochastic collocation methods are some of the popular approaches that have been used in the analysis of such problems.

This work proposes a reduced basis (RB) method for the rapid and reliable evaluation of the statistics of outputs, computed as linear functionals of the solution of stochastic PDEs. We focus on problems where an affine parametrization of the PDE in terms of the uncertain input parameters may be obtained. This particular structure enables us to seek an offline-online computational strategy to economize the output evaluation, and known results regarding *a posteriori* error bounds and adjoint techniques are capitalized to enhance the performance of the RB.

The main novelties of our approach are: (i) the use of high-fidelity solutions in the construction of the basis computed with the hybridizable discontinuous Galerkin (HDG) method—the RB projection is computed to optimally ensure affine parametric dependence; and (ii) the incorporation of a multilevel control variates Monte Carlo method within the RB to achieve enormous savings on the online stage —we solve an inexpensive model selection problem to determine *a priori* the configuration of levels that provides the maximal speed-up with respect to regular Monte Carlo.

The multilevel control variates Monte Carlo is a variance reduction technique that, in the RB framework, corresponds to using in each level a more accurate RB model—with increasing number of basis functions. The finest level can be given by either an extremely faithful RB model or the full PDE, and both approaches are explored and analysed.

We present results for an acoustic wave propagation problem, and verify that the proposed optimality conditions for level selection coincide with the actual optimum. The multilevel Monte Carlo methodology equipped with optimal selection of levels renders online computational savings of several orders of magnitude for the evaluation of output statistics.

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