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An Adaptive Sampling Scheme for Radiation Shielding Calculation

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As we know, the deep-penetration problem in which the penetration rate is underestimated when the thickness is more than about 10MFP has been one of the important and difficult problems in shielding calculation with Monte Carlo Method for several decades. There is a very rich literature on deep-penetration calculation of neutron or gamma. Rong Kong (2008)1 abtained satisfactory results through the adaptive sampling technique. M. LeimdÖrfer(1964)2,3, H. Rief and A. Fioretti(1983)4 gave a particularly good discussion of deep-penetration problem. H.Kahn (1950)5's article probably represents the most general description. In this paper, an adaptive Monte Carlo algorithm [6,7,8] under the emission point as a sampling station for shielding calculation is investigated.

Use (P_m,W_m) ,m=0,1,••• denote the state(position, energy, direction, time) and the weight of the particle from the m-th collision respectively. Use N_m denote the number of sampling of the m-th emission particles, the random walk method can be described as follows:

(1) Sample N_0 particles from the distribution of the source: $(P_{0,n},W_{0,n})_{n=1,2,\ldots,N0}$.
(2) Determine $P'_{m+1,n}$ and $W'_{m+1,n}$,n=1,2,…,N_{m+1}. which is the state of the (m+1)-th emission by using the conventional method from $P_{m,n}$ and $W_{m,n}$ which is the state of the m-th emission of the n-th particle.
(3) $P_{m+1,n}$,n=1,2,…,N_{m+1} will be sampled from the distribution of P_{m+1} which is as follows:

$$\sum_{n=1}^{N_{m+1}} \frac{W'_{m+1,n}}{\sum_{n'=1}^{N_{m+1}} W'_{m+1,n'}} \delta(P_{m+1} - P'_{m+1,n}) \tag{1}$$

(4) Set m=m+1 and return to step 2.

The main advantage of the random walk under the emission point as a sampling station is that the sampling number of the particles N_m ,m=0,1,…, from the m-th emission point station could be controlled by user. That means the best approach could be selected through the adaptive technique.

Let us use D(P) and R(P) denote the particle emission density and its response function of the state P. The variance and the cost can be given approximately as follows, when the N_m ,m=0,1,…, is selected great enough.

$$\sigma^2 = \sum_{m=0}^{\infty} \frac{\sigma_m^2}{N_m}, \qquad C = \sum_{m=0}^{\infty} N_m C_m \tag{2}$$

It is easy to approve that the $\sigma^2 \cdot C$ would reach the minimum value for determined N_0, when

$$N_m = N_0 \frac{\sigma_m}{\sqrt{C_m}} / \frac{\sigma_0}{\sqrt{C_0}} \qquad m=1, 2, \ldots, \tag{3}$$

where σ_m^2 denote the variance under the distribution of $D_m(P)$, C_m denote the computation time through the m-th collision.

An infinite slab of Pb is selected here: the energy of gamma is 4MeV, the source particles are vertical to the surface, the thickness of the slab is selected from 2cm to 42.45cm.

Method-1: the no adaptive method under the emission point as a station based on the exponential transform.Method-2: the adaptive method under the emission point as a station based on the exponential transform. Method-3: the conventional statistical method. Method-4: the importance sampling method under the emission point as a sampling station.

Thicknss （cm）	Method-1		Method-2	
	Penetration rate	variance	Penetration rate	variance
4	2.62E-1	3.49E-1	2.64E-1	5.65E-1
6	1.26E-1	7.30E-1	1.27E-1	7.61E-1
8	5.97E-2	2.07E-0	6.06E-2	1.33E-0
10	2.73E-2	5.97E-0	2.81E-2	2.39E-0
12	1.27E-2	8.01E-0	1.31E-2	4.79E-0
14	5.57E-3	3.28E+1	5.85E-3	8.35E-0
16	2.66E-3	5.65E+1	2.57E-3	1.87E+1
18	1.83E-3	1.43E+2	1.13E-3	1.59E+1
20	6.93E-4	1.03E+3	6.55E-4	5.98E+1

Table 1: **The Comparison of the adaptive with no adaptive**

thickness （cm）	Method-3		Method-4	
	Penetration rate	variance	Penetration rate	variance
8.49	4.63E-2	2.52E-0	4.62E-2	1.13E-0
12.74	8.37E-3	1.36E+1	8.52E-3	2.04E-0
16.98	1.49E-3	7.15E+1	1.56E-3	4.08E-0
21.23	2.60E-4	3.57E+2	2.69E-4	5.94E-0
25.47	2.76E-5	6.44E+2	3.94E-5	9.99E-0
29.72	3.68E-6	9.39E+2	1.02E-5	1.28E+1
33.96	1.47E-6	1.26E+4	6.76E-7	8.58E+1
38.21	2.06E-7	4.45E+4	1.59E-7	1.43E+2
42.45	6.61E-9	1.17E+3	2.03E-8	8.58E+2

Table2: **The importance sampling with conventional method**

It can be found in table 1 that the penetration rate of the two kind of method is matched well. But the variance of the method-2 is lower than method-1 when the thickness of the slab is great than 8cm and the variation of the variance grown with the increasing of the thickness.

From table 2, it is easy to see that the penetration rate of the two kind of method is matched well when the thickness of the slab is not too big. The phenomena which the penetration rate is underestimated in method-3 emerged apparently, when the thickness is greater than 8.49cm, and the underestimation become more serious with the increasing of the thickness.

In the present work, the adaptive Monte Carlo method under the emission point as a sampling station and the related importance sampling technique are derived. The numerical results show that the adaptive method may improve the efficiency of the calculation of shielding and might overcome the under-estimation problem easy to happen in the deep penetration calculation in some degree.

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