11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

A HIGHER-ORDER MULTIDIMENSIONAL UPSTREAM SCHEME FOR THE SIMULATION OF TWO-PHASE FLOWS IN POROUS MEDIA

Paulo R. M. Lyra[1], Márcio R. A. Souza[2], Fernando R. L. Contreras[3] and *Darlan K. E. Carvalho[4]

[1] Department of Mechanical Engineering, UFPE, Av. Acadêmico Hélio Ramos s/n, CEP: 50670-901, Recife, PE, Brazil, prmlyra@padmec.org
[2] Department of Renewable Energy Engineering - UFPB, BR 230, km 21 s/n, CEP: 58059-900, João Pessoa, PB, Brazil, souza2105@gmail.com
[2] Department of Mechanical Engineering, UFPE, Av. Acadêmico Hélio Ramos s/n, CEP: 50670-901, Recife, PE, Brazil, ferlicapac@gmail.com
[3] Department of Mechanical Engineering, UFPE, Av. Acadêmico Hélio Ramos s/n, CEP: 50670-901, Recife, PE, Brazil, dkarlo@uol.com.br

Key Words: *Fluid Flow in Porous Media, MPFA-Diamond, Higher-Order Hyperbolic Scheme, Truly Multidimensional Method.*

Under certain simplifying assumptions, the modeling and simulation of the incompressible and immiscible two-phase flow of oil and water in porous media involves the numerical solution of two different partial differential equations, an elliptic pressure equation with a non-isotropic and non-homogeneous (possibly discontinuous) diffusion coefficient (i.e. the permeability field) and a non-linear hyperbolic (in the absence of capillary and gravity effects) saturation equation, representing a great challenge for traditional numerical methods such as the Finite Element Method (FEM) or the Finite Volume Method (FVM) [1,2,3]. Commercial codes commonly use the classical cell-centered finite difference (Control Volume Finite Differences – CVFD) to discretize the diffusive terms and the First Order Upwind Method (FOUM) to approximate the advective terms arising in fluid flow equations. Despite of its simplicity and apparent robustness, this strategy has its drawbacks, as it is unable to provide consistent approximations for non-orthogonal meshes or full tensor permeability fields. Besides, for flows with high mobility ratios, the conventional FOUM suffers from Grid Orientation Effects (GOE) and poor shock resolution due to excessive numerical diffusion [2,3]. It is interesting to note that, in such context, the use of mesh refinement or naïve higher order methods do not alleviate the GOE, because they are unable to reduce the anisotropic numerical diffusion associated to the upwind weighting of the advected quantity [3]. On the other hand, several efforts have been done in order to reduce or eliminate the mesh dependence in the numerical solution. Truly multidimensional first-order schemes were proposed in a two-phase flow context for node-centered [4] and cell-centered discretizations [5,6]. The higher-order node centered and multidimensional scheme proposed in [7] presents much better results compared to standard low order formulations. In the present paper, in the context of an IMplicit Pressure Explicit Saturation (IMPES) procedure we combine an unorthodox Multipoint Flux Approximation Method based on a Diamond type stencil (MPFA-D) to discretize the diffusive term of the pressure equation [8] with a higher order and Truly Multidimensional Finite Volume Method (TM-FVM) to discretize the advective term of the saturation equation.

In the MPFA-D method, diffusive fluxes on each cell edge are explicitly expressed by two cell-centered unknowns associated to the cells sharing that edge, and two vertex unknowns at the two edge endpoints. These vertex variables are treated as intermediate ones being expressed as linear combinations of the neighboring cell-centered unknowns reducing the scheme to a completely cell-centered one. The scheme is very robust and capable of reproducing piecewise linear solutions exactly in any poligonal meses, by means of a linear preserving interpolation with explicit weights, avoiding the solution of locally defined systems of equations. Besides, The TM-FVM takes into account the flow orientation to construct the numerical fluxes, producing solutions that are less affected by the GOE phenomena and with much less diffusion than the traditional and multidimensional first-order methods available in literature. Furthermore, a robust and accurate Multidimensional Limiting Process (MLP) [9] is employed in order to avoid spurious oscillations that are typical of higher order methods. The MLP presents a much better performance in capturing discontinuous flow features than the other traditional and non-multidimensional limiters. Time integration is performed by a simple Euler Explicit method. In order to verify the proposed formulation, we present results obtained when solving some challenge benchmark problems.

REFERENCES

[1] Carvalho, D. K. E.; Willmersdorf, R. B.; Lyra, P. R. M. A Node-Centered Finite Volume Formulation for the Solution of Two-Phase Flows in Non-Homogeneous Porous Media. *International Journal for Numerical Methods in Fluids*. 2007; **53** : 1197 – 1219.

[2] Durlofsky, L. J. A Triangle Based Mixed Finite Element – Finite Volume Technique for Modeling Two-Phase Flow Through Porous Media. *Journal of Computational Physics* 1993; **105** : 252-266.

[3] Edwards, M. G. Higher-Resolution Hyperbolic-Coupled-Elliptic Flux-Continuous CVD Scheme on Structured and Unstructured Grids in 2-D. *International Journal for Numerical Methods in Fluids*. 2006; **51** : 1059 – 1077.

[4] Hurtado, F.S.V., Maliska, C.R., da Silva, A.F.C., Cordazzo, J.: A quadrilateral element-based finite-volume formulation for the simulation of complex reservoirs. In: SPE Paper 107444-MS presented at the SPE Latin American and Caribbean Petroleum Engineering Conference held in Buenos Aires, Argentina, 15–18 April (2007).

[5] Kozdon, J. E; Mallison, B. T.; Geritsen, G. T. Multidimensional Upstream Weighting for Multiphase Transport in Porous Media. Computational Geoscience 2011; **15**: 399 – 419**.** DOI 10.1007/s10596-010-9211-5.

[6] Eymard, R.; Guichard, C.; Masson, R. Grid Orientation Effects in Coupled Finite Volume Schemes. Journal of Numerical Analysis. 1 – 24; 2012.

[7] Lamine, S.; Edwards, M. G. Multidimensional Convection Schemes for Flow in Porous Media on Structured and Unstructured Quadrilateral Grids. *Journal of Computational and Applied Mathematcs* 2010; **234** : 2106-2117.

[8] Gao, Z. M.; Wu, J. M. A linearity-preserving cell-centered scheme for the heterogeneous and anisotropic diffusion equations on general meshes. *International Journal for Numerical Methods in Fluids* 2010; **67** : 2157-2183.

[9] Park, J. S.; Kim, Chongan. Multi-dimensional Limiting Process for Finite Volume Methods on Structured Grids. Computers and Fluids 2012; **65** : 8 – 24.