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SOME ADVANCES ON ANCHORED ANOVA EXPANSION FOR HIGH ORDER MOMENTS COMPUTATION

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The Analysis of Variance (ANOVA) expansion is an elegant and very useful way to represent a high dimensional multivariate function $f(\mathbf{x})$, for instance, when estimating the sensitivity indices via variance-based approaches. For independent random inputs, ANOVA from standard definition consists in a unique orthogonal decomposition of $f(\mathbf{x})$. Each component function provides its best approximation to $f(\mathbf{x})$ in a least-square sense. From computational point of view, standard orthogonal ANOVA can be very expensive when encountering very high dimensional problems and complicated multivariate functions. Indeed, the drawback of standard ANOVA consists in the need to compute the high-dimensional integrals (often requiring Monte Carlo type sampling methods). Even the zeroth-order component function requires a full-dimensional integration in the stochastic space. Alternatively, anchored ANOVA decomposition [2, 1, 6, 5] gives an efficient way for the numerical evaluation of component functions in ANOVA expansion. As a consequence, the estimation of mean and variance become feasible for real engineering problems. In particular, [5] presents some adaptive criteria as dimension reduction techniques, which can be applied to problems with a very high number of stochastic variables. One main drawback appears in such a decomposition: the accuracy of approximation is found to be very sensitive to the choice of the "anchor point". [4, 3] show that a bad choice of the anchor point can lead to an unacceptable approximation error. This talk analyzes the reason of this sensibility to anchor point, and proposes to use the covariance decomposition of the variance that has the capability of evaluating very accurately the output variance in the framework of anchored ANOVA. We then propose to employ the covariance-based sensitivity indices to study the relative significance of independent input variables. We then extend this technique to the general case with the aim of evaluating high order statistical moments: skewness and kurtosis. In particular, we will show the sensibility problem related to the anchor point can be explained and successfully avoided by the proposed strategy. Numerical experiments show that our approach provides converged results for all moments when considering academic problems. On the other hand, it generally gives more accurate results than classical method, when using truncated approximation for high dimensional problems. An engineering test case will be presented which involves the computation of the radiative heat flux at a distance corresponding to the stand-off distance for the ERC capsule.

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