

## REDUCED INTEGRATION WITH HOURGLASS STABILIZATION – ISSUES OF STABILITY AND ROBUSTNESS

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### ABSTRACT

In the past decades much research has been invested to cure the well-known undesirable stiffening effects of low-order finite elements in solid mechanics. A large part of effective concepts against these so-called locking phenomena, e.g. shear and volumetric locking, are based on multi-field variational functionals. The latter lead to mixed finite element formulations and, if further assumptions are applied, to finite elements using reduced or selective reduced integration. Many formulations developed in recent research show highly satisfactory behaviour regarding deformation and stress fields.

Nevertheless, one important issue is usually neglected. On the one hand, in contrast to linear elasticity, the uniqueness of a solution cannot be required in general, since singular solutions such as bifurcation points might become physically relevant and should be displayed by a powerful numerical method. On the other hand, the use of multi-field variational functions introduces additional variables (e.g. the enhanced strain) into the formulation which enormously increases the complexity of the stability issue [1] and eventually leads to artificial bifurcation points [2].

In the present contribution, the problem is investigated in the context of reduced integration techniques. The split of the tangential stiffness matrix into one part ( $K_0$ ) which is obtained by summing up the contributions from a reduced number of Gauss points and another part ( $K_{stab}$ ) frequently termed hourglass stabilization matrix is used to make a prognosis whether non-physical instabilities might show up. This analysis can be performed independently of the special finite element formulation of interest. The only requirement is the split of the tangential stiffness matrix into  $K_0$  and  $K_{stab}$ . We investigate reduced integration techniques based on different variational principles as well as solid-shell and solid-beam formulations. Not only the issue of stability is analysed in this way but also the ability of different element formulations to undergo severe mesh distortion.

### References:

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