

## Evaluation of time integration schemes in elastodynamics using numerical amplification matrices

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In this work we show the convenience of obtaining and using numerical amplification matrices in the analysis of time integration schemes in elastodynamics. Numerical amplification matrices may be directly obtained computing the response to several single degree of freedom systems obtained with the integration algorithm at hand [1]. Therefore, properties of the algorithm, as for example the spectral radius, may be readily obtained without the need for lengthy mathematical manipulations. Furthermore, in contrast with analytical, theoretical amplification matrices, numerical amplification matrices show what the computer is actually doing in the real implementation using non-exact arithmetic. Then, computing numerical amplification matrices may give some extra information that can be “hidden” in the formulation and that can remain unnoticed using the traditional analytical calculation of the amplification matrix.

The analysis of algorithms using numerical amplification matrices gives an interesting insight on the proper minimum dimension of the amplification matrix as related to the structure and to the initial conditions to be prescribed to initialize the algorithm. For example, the well-known overshooting phenomenon that is present in some methods as the Wilson- $\theta$  or HHT methods [2], [3] can be detected and studied using numerical amplification matrices. A proposal to minimize such behaviour is also given in this work.

### REFERENCES

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