11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

GRADIENT DAMAGE MODELS AND BRITTLE FRACTURE

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Key words: *Fracture, Damage, Variational Methods, Stability, Finite Elements, Unilateral Problems, Thermal Shock*

Damage models are commonly used to predict the failure of brittle structures. However, how such models relate to brittle fracture is still a generally open topic. We present recent analytical and numerical results pertaining to the evolution of a general class of isotropic *damage models with a gradient term in the damage variable* and an internal length ℓ [1–4] and how they can relate to Griffith–like theories.

Our approach is *phenomenological*: instead of giving a microscopic interpretation of the damage theory, we study the properties of *quasi-static irreversible rate-independent evolutions* [7] based on two constitutive assumptions: (i) *stress-softening*, (ii) *finite dissipation* at complete failure. With additional restriction on the class of models, one can recover the existence of a non-vanishing elastic limit and link it to the fracture toughness through the internal length. In the one-dimensional case, this analysis can be made rigorous: crack nucleation is due to the loss of stability of homogenous solutions which, for sufficiently long structures, takes place at the elastic limit. The response exhibits a snap-back toward localized responses. The dissipated energy of the fully localized solution, seen as a regularized representation of a crack, can be linked to the critical elastic energy release rate.

This framework extends seamlessly in 2D and 3D settings and naturally leads to a numerical solution scheme based on the solution of incremental non-convex minimization problems under irreversibility constraint on the damage variable. These problems are solved using finite elements discretization in space including the displacement and damage fields as nodal variables, and an alternate minimization strategy [3]. We illustrate the capabilities of the proposed approach to predict the morphogenesis and propagation of complex crack patterns with results obtained for a thermal shock problem [5, 6] (see

Figure 1).

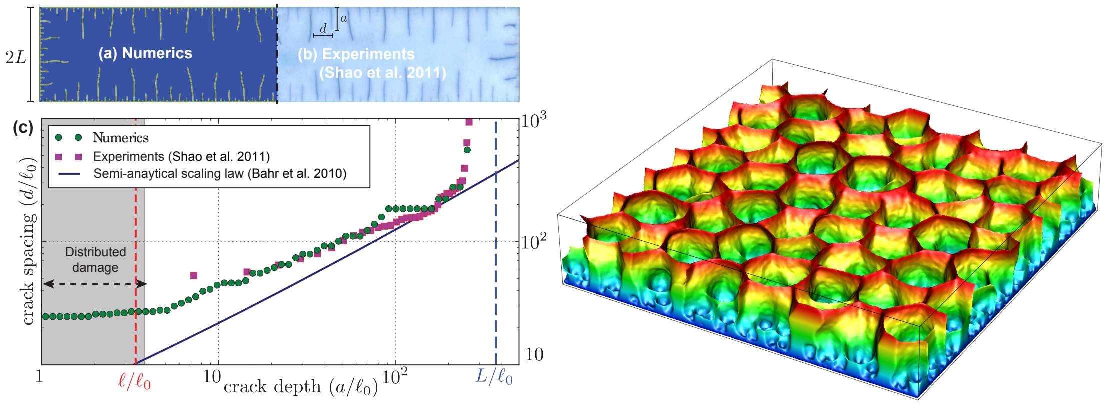

Figure 1: Numerical results on thermal shock cracks. *Left:* Full scale 2D numerical simulation with (a) Damage field from the numerical simulation (b) Experimental results from [8, FIG. 5(d)]. (c) Average crack spacing d as a function of their depth a for (a) and (b). *Right:* Three-dimensional fracture pattern color-coded by the distance from the bottom surface where the thermal shock is applied.

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