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A FACE-BASED SMOOTHED FINITE ELEMENT METHOD FOR HYPERELASTIC MODELS AND TISSUE GROWTH

Minh Tuan Duong[*,1,2] **and Manfred Staat**[1]
[1] Aachen University of Applied Sciences Heinrich-Mußmann-Str. 1, 52428 Jülich, Germany
e-mail: {duong, m.staat}@fh-aachen.de
[2] Hanoi University of Science and Technology, No.1 Dai Co Viet Road, Hanoi, Vietnam

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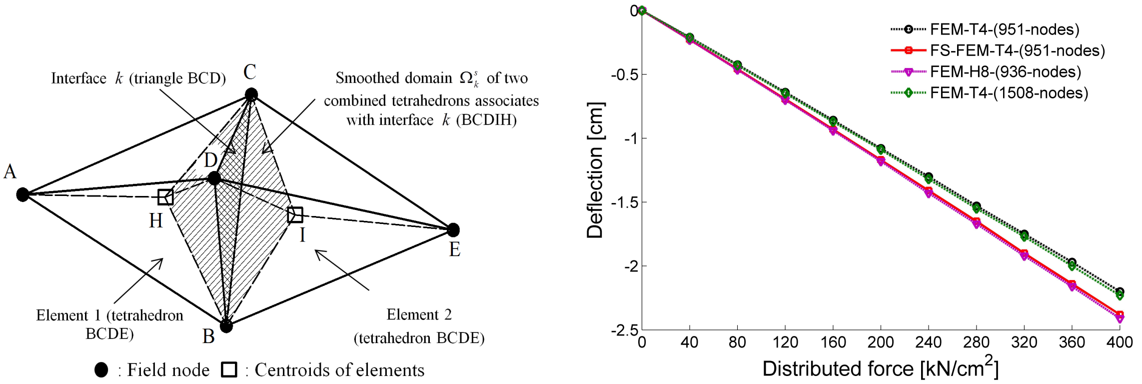
To simulate very complex 3D shapes of soft tissues (e.g. human organs), the Finite Element Method (FEM) is often adopted by using the T4 linear tetrahedral element (FEM-T4). However, in this approach there are still existing crucial shortcomings of the method for problems of solid mechanics such as the well-known overly stiff behavior, poor stress solution, and volumetric locking in nearly incompressible cases. To overcome these disadvantages, this paper presents the Face-based Smoothed Finite Element Method (FS-FEM) using T4 elements (FS-FEM-T4) applied to physically and geometrically nonlinear problems with improved accuracy. Concretely, the FS-FEM is one of the Smoothed Finite Element (SFEM) models [1]. S-FEM are methods in between the FEM and the mesh-free methods [2]. In the field of biomechanics, the SFEM is still relative new because up to date there have been only few research using simple isotropic hyperelastic materials. We have implemented the FS-FEM into the open source software Code_Aster [3] for large scale biomedical applications. The principal idea of the FS-FEM is to formulate a strain field as a spatial average of the standard strain measure. To this end, the elements are divided into smoothing domains associated with the faces of the tetrahedral elements over which the strain is smoothed, see Figure 1a. Integration over the element is now transformed to boundary integration of the smoothing cell using the divergence theorem. Since the stiffness matrix is built based on boundary integration, no requirements of derivatives of shape functions and isoparametric mapping are needed. Consequently, the FS-FEM method produces several advantageous properties such as improved accuracy and superconvergence, relative insensitivity to mesh distortion.

To verify the implementation of the FS-FEM model, a number of test cases are conducted in which the method is employed for linear cases and highly nonlinear problems. In the nonlinear case of solving a 3D cantilever beam subject to a regular distributed load with the material formulated by the strongly anisotropic hyperelastic material models by Duong et al. [4]. This can be considered the first application in which the FS-FEM is applied to hyperelastic models with strong anisotropy (e.g. artries). The tip deflection is recorded



(a) Smoothing domain created on an interface of two neighboring T4 elements

(b) Displacement-force curve of the free end of the 3D cantilever beam

Figure 1: Smoothing domains creation of the FS-FEM and numerical results

and shown in the Figure 1b. It is clearly seen that FS-FEM solution has higher accuracy and its accuracy and convergence can be compatible to those of the standard FEM with 8-node hexahedral element (H8) (FEM-H8) using the same number of nodes. Numerical results of the FS-FEM for linear and nonlinear problems noticeably show its advantages in improving accuracy of distorted meshes. In addition, it is also emphasized that tissue growth models [5] are first analyzed by the FS-FEM. To do that, internal variables of the growth models are modified properly and the implicit Euler integration is implemented to solve the equation system for growth simulation. This is an important process of living tissues subjected to external load, which is considered as stimulus factor in their lives. In conclusion, the computational efficiency of the FS-FEM is found better than that of the FEM. The FS-FEM not only brings about higher accuracy but also relative insensitivity to volumetric locking if it is combined with a Node-based Smoothed Finite Element (NS-FEM) called FS/NS-FEM model. This is a very promising trend for applying the FS-FEM in biomechanics in which incompressible materials are often required.

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