

SCATTERING OF SEMI-CYLINDRICAL GAP AND MULTIPLE SHALLOW-BURIED CAVITIES AND INCLUSIONS BY SH-WAVE

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In natural medium, engineering composite materials, earthquake engineering, it can be found that there are some gaps at the edge of the matrix, and shallow-buried cavity or inclusion structure near the gaps. When the matrix is impacted by dynamic load, scattering field will be produced because of the cavity or the inclusion, and it could cause dynamic stress concentration at the edge of the cavity or inclusion. Dynamic stress concentration could greatly decrease the bearing capacity of the matrix, and reduce the service life of the matrix. Researchers solved some simple problems by analysis and numerical methods[1-2].

The problem of scattering of SH wave by multiple shallow-buried cavities and inclusions is one of the important and interesting questions for the latest decades. There are lots of materials obtained by theoretical research and damage investigation. This problem is complicated, because there are many factors influenced.

In recent years, there have been many researches on elastic wave propagation in phononic crystals (PCs). Full band gaps have caught much attention. The full band gaps are frequency domains in which propagation of sound wave are forbidden, The solution of interaction of semi-cylindrical gap, multiple cylindrical cavities and inclusions can be useful in the field of PCs, too.

The model is shown as Fig.1, an elastic semi-space containing semi-cylindrical gap and multiple shallow-buried cavities and inclusions which bears anti-plane SH wave. Generally, the displacement in elastic semi-space is expressed as $W(x, y, t)$. The displacement in inclusions are expressed as $W_k(x, y, t)$ ($k = 1, 2, \dots, M$).

The boundary conditions can be expressed as below:

$$\begin{aligned} \tau_{rz} = 0(\text{where } |z| = R_0), \quad \tau_{rz} = 0(\text{where } |z - c_j| = R_j, j = 1, 2, \dots, N), \\ W = W^{(sm)}(|z - C_k| = R'_k, k = 1, 2, \dots, M), \quad \tau_{rz} = \tau_{rz}^{(sm)}(|z - C_k| = R'_k, k = 1, 2, \dots, M). \end{aligned}$$

The incident wave and reflection wave can be written as $W^{(i)}$ and $W^{(r)}$. The scattering wave incited by semi-cylindrical gap and by multiple shallow-buried cavities and inclusion can be written as $W^{(Os)}$, $W^{(is1)}$, $W^{(is2)}$. The displacement in the inclusion can be written as $W^{(sm)}$.

Therefore, the wave field W can be written as $W = W^{(i)} + W^{(r)} + W^{(Os)} + W^{(is1)} + W^{(is2)}$.

The wave field W must satisfy the stress or displacement condition on semi-cylindrical gap and multiple shallow-buried cavities and inclusions, so by using the method of transferred coordinate, substituting the wave field W to the boundary conditions, it can be obtained that

$$\sum_{n=0}^{\infty} A_n \phi_{mn}^0 + \sum_{j=1}^N \sum_{n=-\infty}^{\infty} A_n^j \phi_{mn}^j + \sum_{k=1}^M \sum_{n=-\infty}^{\infty} B_n^k \phi_{mn}^k = \phi_m, \quad \sum_{n=0}^{\infty} A_n \phi_{mn}'^0 + \sum_{j=1}^N \sum_{n=-\infty}^{\infty} A_n^j \phi_{mn}'^j + \sum_{k=1}^M \sum_{n=-\infty}^{\infty} B_n^k \phi_{mn}'^k + \sum_{n=-\infty}^{\infty} C_n^m \phi_{mn}' = \phi_m',$$

$$\sum_{n=0}^j A_n \phi_{mn}^0 + \sum_{j=1}^N \sum_{n=-\infty}^{\infty} A_n^j \phi_{mn}^j + \sum_{k=1}^M \sum_{n=-\infty}^{\infty} B_n^k \phi_{mn}^k + \sum_{n=-\infty}^{\infty} C_n^m \phi_{mn} = \phi_m.$$

By multiplying both sides of former equations with $e^{-im\theta_m}$ and integrating in interval $[0, \pi]$ or $[-\pi, \pi]$, it can be obtained a set of infinite algebraic equations for determining the coefficients A_n^j and B_n^k . Then the total wave field W of this problem can be obtained.

In theory of elastic wave motion, dynamic stress concentration is a danger factor. In this paper, we pay attention to a representative kind of models, which is shown as Fig.2, and discuss variety of Dynamic stress concentration factor (DSCF) at the cavity or inclusion edge. From Fig. 3 it can be found that the influence among the gap, cavities and inclusions is very complex.

Based on this solution, the problem of interaction of semi-cylindrical gap, multiple cylindrical cavities and inclusions and linear cracks in semi-space could be investigated further.

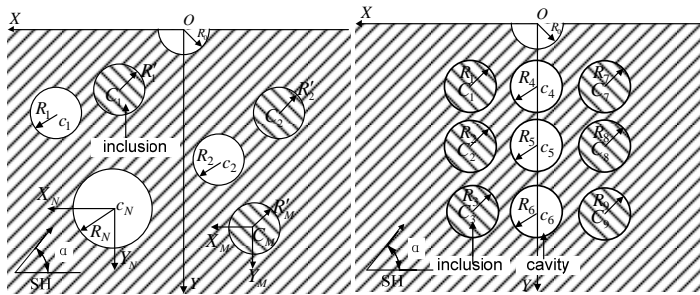


Fig. 1. Model of problem

Fig. 2. Calculating model

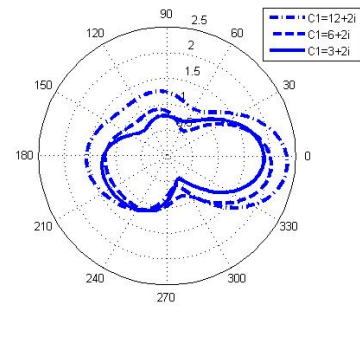


Fig. 3. Influence of semi-cylindrical gap to DSCF at the inclusion edge

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