11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

CONSTITUTIVE AND COMPUTATIONAL MODELLING OF THE EFFECT OF FIBRE BENDING RESISTANCE IN REINFORCED ELASTOMERS

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Key Words: *Hyperelasticity, Anisotropy, Fibre composite, Couple Stress Theory.*

When dealing with the fibre-reinforced composite it is a common approach to replace the heterogeneous model with the homogeneous one for the sake of computational efficiency. Conventional modelling of fibre reinforced solids employs fibre vector formulation [1]. It means that unit vector field defines the direction of the fibre family and is treated as a constitutive variable. Then the bending stiffness of the resulting homogeneous model is defined by the effective tensile stiffness of the heterogeneous one. This, in general, does not give the valid representation of real behaviour.

The homogenous model which precisely corresponds to the heterogeneous one in relation to the bending behaviour can be constructed using couple stress theory.

To incorporate fibre bending stiffness into the homogenous model, Spencer and Soldatos [2] proposed the strain-energy density function that depends on deformation, fibre direction and gradients of the fibre direction in the deformed configuration. Thus, the fibre curvature is included in the governing equations.

We followed theoretical framework proposed in [2] with some modifications and developed verifiable computational implementation of the suggested model. As a result, the effect of the individual fibres with bending stiffness is "smeared out" uniformly throughout the section of the homogenous model.

The following form of the strain energy density was chosen:

$$W = k_1\left(\bar{I}_1 - 3\right) + k_2\left(\bar{I}_4 - 1\right)^2 + k_3\bar{I}_6 + \frac{1}{d}\left(J - 1\right)^2,$$

where $\bar{\boldsymbol{I}}_1$ is invariant of deviatoric Cauchy-Green deformation tensor that accounts for the isotropic hyperelastic matrix, $\bar{\boldsymbol{I}}_4$ is invariant that accounts for the stretching of fibres, $\bar{\boldsymbol{I}}_6$ relates to the bending of fibres and the last term accounts for volumetric deformation.

Appropriate finite element formulation was derived and finite element solver was developed. Test computations show that the bending stiffness can be regulated with the constant k_3. On the basis of the linearized constitutive relations of the aforementioned theory, the analytical procedure for the evaluation of the constant k_3 is offered.

Acknowledgements. This work was supported by Czech Science Foundation project number 13-16304S and by faculty project No. FSI-S-14-2344.

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