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DUAL ERROR BOUNDS FOR KIRCHHOFF PLATES

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Dual analysis techniques [1] work on a pair of approximate solutions, one compatible, the other equilibrated, allowing to obtain bounds of their errors, either global or local [2, 3].

For most applications of the Finite Element Method in linear elasticity, the challenge involved in using these techniques is related to difficulties in obtaining solutions that are strictly equilibrated. Most equilibrium finite elements present numerical instabilities, which may hinder the solution process. Therefore most approaches prefer to compute the equilibrated solution using some recovery process, avoiding the problems of a large singular system and taking advantage of the good performance of the compatible finite element models.

Nevertheless, bending of plates modelled using the Kirchhoff theory are an exception to this rule. On one hand, in displacement formulations, using the transverse displacement and the components of its gradient as nodal degrees of freedom normally leads to non compatible approximations of the transverse displacement, because they are not C_1 continuous. Most approaches used to overcome this difficulty, *e.g.* Discrete Kirchhoff elements, fulfil the requirements for convergence, but are not strictly compatible. Though it is possible to compute estimates of the error from such solutions, they are no longer bounds, unless additional corrective terms, in general dependent of the (unknown) exact solution, are considered.

On the other hand a general degree hybrid equilibrium element, which can be regarded as a generalisation of Veubeke's [4] and Morley's triangles [5], has been recently developed [6]. This element is free from spurious kinematic modes, allowing for its use with the same ease, actually with the same code, as conventional displacement elements.

In this communication we will review a classical family of approximations for Kirchhoff plates, which lead to strictly compatible approximations, the TUBA [7] elements.

Then we present the fundamentals of the aforementioned Hybrid Equilibrium formulation [6], discussing the basis for its implementation: the approximation of the moment fields, the selection of the hybrid boundary approximations, the enforcement of equilibrium at the sides and vertices of the elements, and the kinematic stability of the resulting element. The application of local recovery techniques, either obtaining compatible solutions from the equilibrated ones or vice-versa will also be considered.

We complete the communication by presenting numerical tests where, from the approximations provided by both types of solutions, bounds of the global error in energy are obtained.

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