

## End effect on fluid permeability of particulate layers

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The glycocalyx layer covering vascular endothelial surfaces (EGL) has received remarkable attention owing to its significant roles in various physiological functions. One of these functions is a molecular sieve when macromolecules are transported to and from the circulating blood and the surrounding tissues. Intensive studies on the transport property of the EGL started since detailed structural analyses revealed the presence of three-dimensional fibrous meshwork within the EGL with fibrous strands projecting normally to the surface of the vessel wall in characteristic spacings of about 20 nm<sup>[1]</sup>. Based on this observation, several research groups adopted a simplified mathematical model for the EGL which consists of a periodic array of circular cylinders representing core proteins, and showed successfully that the center-to-center spacing of 20 nm between adjacent cylinders and the cylinder diameter of 10-12 nm provide just the size regime to account for the observed molecular filtering of vessel walls<sup>[2,3]</sup>. However, in their analyses, the thickness of the EGL was assumed to be infinite or the end effect was neglected. Recent studies have shown that the thickness of the EGL and its constitution vary by the blood vessel site as well as physiological and pathological conditions. In addition, modelling the core proteins as circular cylinders may be too simplified. In the present study, we consider the fluid permeation through particulate layer with finite thickness. By theoretical analysis of fluid permeability of the layer where spherical particles are arranged with arbitrary regular spacings and finite thickness, effects of the end (i.e. the boundary between the fluid phase and the particulate layer) and the interparticle distance on the permeation characteristics are investigated.

The particulate layer used for the analysis is shown in Fig. 1. Spherical impermeable particles with radius  $a$  are arranged at the corner of the cuboid array which has a finite thickness  $L$  in the  $z$  direction and infinite in the other  $x$  and  $y$  directions. The interparticle distance in the  $x$ ,  $y$ , and  $z$  direction is represented by  $l_x$ ,  $l_y$ , and  $l_z$ , respectively. We set  $l_x=l_y$  in the current study. We consider an incompressible Newtonian fluid flow in the  $z$  direction. The Reynolds number for the flow through the EGL is so small that the Stokes flow is assumed. Theoretical calculation of the permeability was performed on the basis of the Stokesian dynamics approach which enables to describe the effect of the presence of multiple particles on the flow field<sup>[4]</sup>. Two-dimensional Ewald summation technique was applied for the periodic boundary in infinite  $x$ ,  $y$  directions<sup>[5,6]</sup>.

Figure 2 shows the permeability profile of the particulate layer in the  $z$  direction. The permeability is defined as a ratio of the flow rate to the pressure difference, and in Fig. 2 its local value is represented by the ratio relative to that for three-dimensionally infinite system under the same condition. The positions  $z/a=0$ ,  $L/a$  in vertical axis correspond to both ends of the layer. It is observed from Fig. 2 that the permeability is different at the ends while the  $k_{2D}/k_{3D}$  shows unity at a large part inside the layer. This is interpreted as the end effect on the

permeability. Although the previous study has reported that the end effect depends on the particle positional relationships<sup>[5]</sup>, decreases in permeability are observed at both ends of the particle layer used in the current analysis. It is inferred that the presence of the surrounding particles forms the flow field which increases the permeability because the permeability decreases in the vicinity of the end where the neighboring particles exist only on one side. A comparison between (a) and (b) shows that the end effect increases with decreasing  $lz$ . It indicates that the close distance between particles in the flow direction causes the increase of the end effect since the particle arrangement at the end is more anisotropic. A comparison of (a) and (c), on the other hand, indicates that the end effect becomes definite for greater  $lx$  ( $=ly$ ). This would explain that the end effect is inhibited by the disturbed flow field in the presence of particles aligned normal to the mean flow direction. The same tendency can be observed in the results for various  $lx$  ( $=ly$ ),  $lz$  conditions.

Figure 3 shows the total permeability of particulate layer with various bed thickness  $L/a$ . As the thickness increases,  $k_{2D}/k_{3D}$  approaches unity. This suggests that the end effect can be negligible if the layer thickness is large enough. The end effect is more substantial as the thickness is smaller. From comparison among (a), (b) and (c), the magnitude of the effect depends on the local permeability variance at the end which is observed in Fig. 2. To summarize, it is found that the interparticle distance in the flow direction and that in the other directions affect the flow field near the end of the layer in a different manner. The total permeability of finite layers is more affected by the layer thickness for thinner layer.

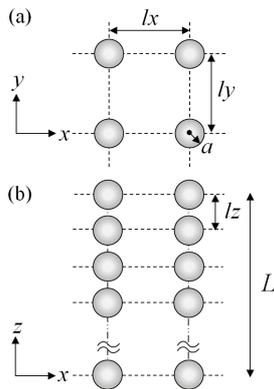


Fig. 1 Particulate layer  
(a)  $x$ - $y$  and (b)  $x$ - $z$  plane

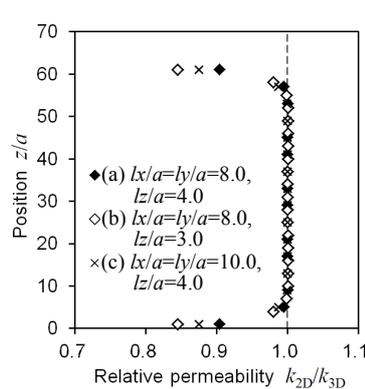


Fig. 2 Permeability profile in  $z$   
direction ( $L/a=62.0$ )

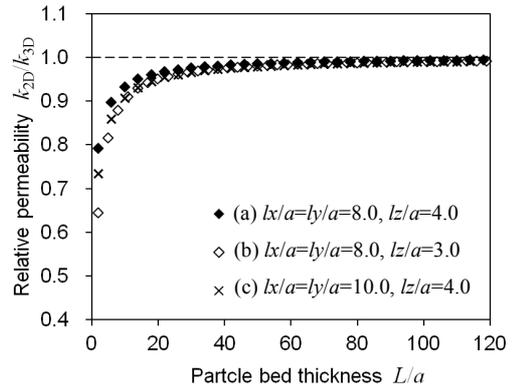


Fig. 3 Total permeability against  
layer thickness  $L$

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