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# PROJECTION-BASED ROMS FOR PARAMETRIZED OPTIMIZATION PROBLEMS CONSTRAINED BY PDES: RESULTS AND APPLICATIONS

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Parametrized optimization problems governed by PDEs occur in several applied problems ranging from optimal control and/or design problems to inverse identification problems. Parameters may arise from physical coefficients, the geometrical configuration, or the control/design variables themselves. Solving these problems is rather challenging because of their (most often) large scale and/or their iterative nature.
For instance, when using a descent method for the numerical optimization, the underlying state PDE has to be solved many times until the minimum of the cost functional is approached. Should we be interested to the solution of an optimal control problem for many input conditions, characterizing either the state problem or the cost functional, computational costs would be even higher [2].

Projection-based reduced-order models (ROM) provide efficient strategies to tackle parametrized optimization and control problems, thanks to Offline/Online computational stratagems, sharp a posteriori error bounds, and the use of low-dimensional approximation spaces [1]. Two different classes may be identified, depending upon the role of the parameters:

- when dealing with control parameters, we can exploit either a direct-ROM or an adjoint-ROM. In the former, we build a reduced basis (RB) approximation of the state problem, whose online evaluation can be realized by any numerical optimization procedure, thus leading to a "reduce-then-optimize" approach [3]. In the latter, we consider two distinct ROMs for the state and the adjoint problem, thus leading to an "optimize-then-reduce" approach.

- instead, when dealing with distributed/boundary control functions and parameters directly expressing some features of the state problem, but not the control function itself, a more general approach can be envisaged, leading to a simultaneous reduction of both state and control spaces. In this case, a global ROM for state, adjoint and control variables allows a further enhancement of the computational speedup.

In both cases, we can provide suitable a posteriori error bounds, in order to certify the solution of the optimization problem obtained with a reduced basis method. We present some numerical test cases, assessing both the approximation properties and the computational efficiency of these methods, and draw some useful guidelines for their employment in more complex applications [4].

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