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AN ALWAYS ACCURATE AND SOMETIMES EXACT DISCRETIZATION OF THE CONVECTION-DIFFUSION EQUATION IN 1D AND 2D

A. Pascau[1] and F. Alcrudo[2]

[1] Fluid Mechanics Group and LIFTEC, CSIC-University of Zaragoza, María de Luna 3, 50018, Zaragoza, Spain, pascau@unizar.es

[2] Fluid Mechanics Group and LIFTEC, CSIC-University of Zaragoza, María de Luna 3, 50018, Zaragoza, Spain, alcrudo@unizar.es

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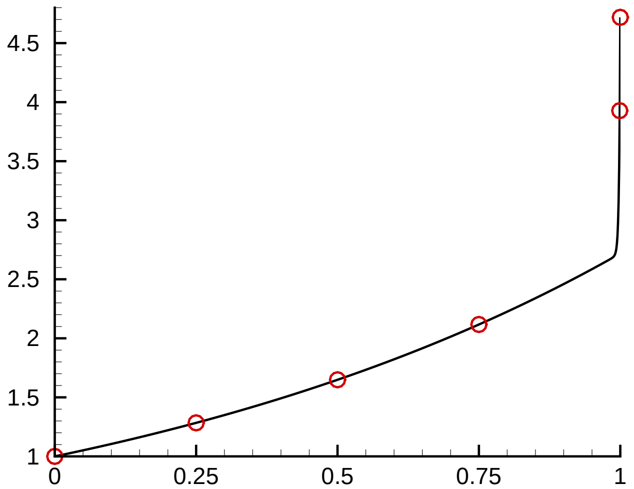
This paper presents a generalization of the ENATE procedure [1] thereby an exact discretization of a 1D convection-diffusion equation with piecewise-constant coefficients and arbitrary source was obtained. This generalization involves arbitrarily varying convective and diffusive coefficients. The whole procedure consists in transforming the original equation with source term into another one source-free for which an exact solution is known if the convective flux is piecewise-constant. If it is not, we can still get an exact solution if a certain integral has an analytical primitive.

In 2D flows the situation is slightly different. Even if there is no an explicit source term one has to split the equation in two in order to be able to adopt the above mentioned procedure. In two dimensions flux gradients in one direction act as sources in the convection-diffusion in the other direction, in such a way that the 2D convection-diffusion equation converts in two 1D ones. When dealing with two dimensions the derivatives and integrals of the solution gradients are required in a given interval. To obtain an analytical expression that can be later integrated analytically (if possible) and then evaluated numerically at the nodes, an interpolant of the discretized solution is called for. Several polynomials were checked as interpolants and we finally opted for Hermite polynomials that have adequate properties of monotonicity.

An example of the variable-coefficient 1D cases we consider is that proposed by Tian et al. [2]

$$\frac{d\phi}{dx} = \epsilon(1+x)\frac{d^2\phi}{dx^2} + e^x[1-\epsilon(1+x)] \tag{1}$$

in the interval $[0,1]$. This equation presents a steep boundary layer near 1 as ϵ goes to zero. The details of the original ENATE derivation are in the paper mentioned, the extended procedure follows the same pathlines. Here we will only show results of this variable-coefficient case. We used four internal nodes and two additional nodes at the boundaries where Dirichlet boundary conditions taken from the exact solution were implemented.



In the figure the exact solution is shown as a solid line. As seen, we can get an exact solution with four internal nodes, in fact we could have obtained an exact solution with just one node. This is a particular case for which all integrals involved are analytical but this not always the case, in this latter situation the solution obtained is very accurate but not exact.

REFERENCES

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