

## Isogeometric Analyses of Beams and Shells

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### 1. INTRODUCTION

Isogeometric Analyses (IGA) [1] using NURBS functions enable a smooth surface description with higher inter-element continuity which opens up new possibilities in the analysis of thin-walled structures such as beams, plates and shells. Most important advantage is the straightforward implementation of classical theories, which require  $C^1$ -continuity, e.g. Euler-Bernoulli beam and Kirchhoff-Love shell formulations, as demonstrated in [2]. Based on these “simplest” models shear deformable theories, like Timoshenko beam and Reissner-Mindlin shell formulations, can be formulated in an elegant way [4]. In that contribution a hierarchic set of isogeometric shell finite elements was developed, where shear and curvature-thickness locking (in the case of 3D-shells) has been removed by a pure displacement formulation. A current challenge is the adoption or extension of these ideas in order to also remove membrane locking avoiding additional elaborate methods.

### 2. HIERARCHIC FORMULATION OF BEAMS AND SHELLS

The hierarchic concept will be first explained in the simplest case of plane straight beams. We look at the kinematic equations of a shear deformable Timoshenko beam.

$$\kappa = \varphi \tag{1}$$

$$\gamma = w' + \varphi \tag{2}$$

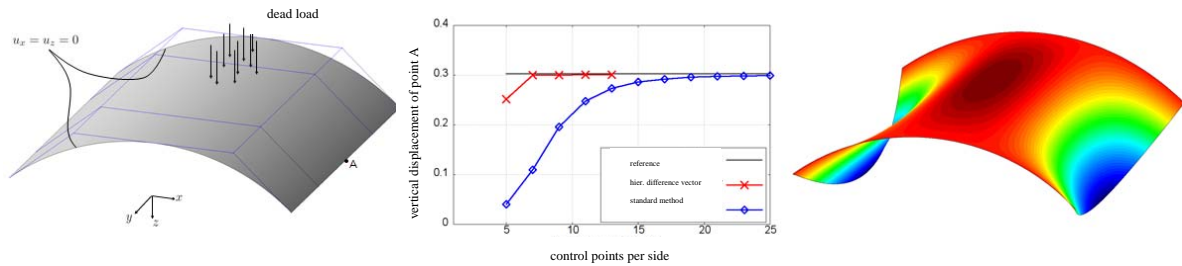
Eq. (1) describes the curvature  $\kappa$  and (2) denotes the shear strain  $\gamma$ . Instead of discretizing deflection  $w$  and total rotation angle  $\varphi$  in a standard way, shear angle  $\gamma$  representing an incremental rotation is introduced as primary variable in addition to  $w$

$$\kappa = -w'' + \gamma \tag{3}$$

$$\gamma = \gamma \tag{4}$$

It is apparent that this alternative discretization scheme yields second derivatives of  $w$ , what requires  $C^1$ -continuity of the shape functions. Eqs. (3), (4) can be identified as classical kinematic equation of the Euler-Bernoulli beam problem enriched by a shear strain  $\gamma$ . In the thin limit  $\gamma$  yields zero, what can be easily satisfied by (4); thus this formulation is free from shear locking without any further treatment. The extension to curved beams and shells is straightforward; it also leads to a hierarchic split of bending and shear free from shear locking. Stepping into the area of curved elements brings in an additional geometric stiffening phenomenon denoted as membrane locking; it is present for shear free as well as shear deformable formulations and seems to be the most crucial problem for low order elements. The inability of representing pure inextensional deformations results from the coupling of membrane and bending action in curved members; within the IGA it has been remedied by

additional methods like the Discrete Strain Gap (DSG) approach ([3],[4]) or mixed approaches ([4],[5],[6]) yielding fully locking free isogeometric beam and shell finite elements. The subsequent Figure addresses the classical benchmark problem of the Scordelis-Lo roof under dead load. The corresponding problem description can be found in [1] or [4]. A standard Reissner-Mindlin formulation is compared to the hierarchic Reissner-Mindlin shell which is enhanced by a mixed approach for the membrane part. The superior behavior of this extended formulation can be identified from the convergence curve for the vertical displacement of point A.



However due to the higher continuity within the IGA these methods have to be applied on patch level, which in general destroys the sparse structure of the equation system.

### 3. CONCLUSIONS AND OUTLOOK

The hierarchic concept has been proven to be an elegant method for the development of isogeometric beam and shell formulations free from transverse shear locking without any further treatment. However one has to realize that membrane locking is still present for low order elements on all hierarchic levels. Currently this defect is removed by extended formulations, such as the DSG or mixed methods ([4],[5],[6]), which are rather expensive. Therefore the present study focuses on the development of a more efficient approach; it is expected that the work, which is still in progress, eventually leads to a formulation which is as elegant as the above described hierarchic concept preventing shear and curvature thickness locking.

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