11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

AN ERROR ESTIMATOR FOR RECOVERED FIELDS IN LINEAR ELASTICITY: TOWARDS HIGH PERFORMANCE H-ADAPTIVE FINITE ELEMENT ANALYSIS

J.J. Ródenas*, E. Nadal, M. Tur and F.J. Fuenmayor

Centro de Investiación en Tecnología de Vehículos (CITV)
Universitat Politècncica de València, Camino de Vera, s/n, E-46022, Valencia, Spain
e-mail: jjrodena@mcm.upv.es, enadalsoriano@gmail.com, {matuva, ffuenmay}@mcm.upv.es

Key words: *Finite element method, error estimation, recovered solution, h-adaptive refinement*

In the context of the linear elasticity problem, the quality of Finite Element Analyses (FEA) has been traditionally evaluated using estimations of the error in energy norm of the Finite Element (FE) solution. One of the techniques used to evaluate the error in energy norm is based on the use of an enhanced or recovered solution obtained form the FE solution. This recovered solution is of a higher quality than the FE solution and therefore it would be interesting to use it as the output of the FE program. However this would require error estimation and error bounding techniques to evaluate its accuracy. However, so far, we can only find a few works in this field [2, 1], which involve a high computational cost. In this work we propose the use of a displacement recovery technique, so-called SPR-CD [3, 4], that by means of a set of constraints imposed at patch level provides both, an improved kinematically admissible recovered displacement field and also an almost-statically admissible stress recovered field. The recovered stress field is continuous but suffers from a lack of internal and boundary equilibrium. An expression to evaluate the exact error in energy norm of the recovered solution can be found in [4]. In order to evaluate the quality of the recovered solution, in this work we introduce an *a-priori* upper bound of the error in energy norm of the recovered stress field, a numerical strategy to evaluate the required constant and the use of the following heuristic error estimator for the recovered stress field introduced in [4]:

$$\mathscr{E}_3^* = \sqrt{\int_\Omega |(\mathbf{s}_\sigma^*)^T \mathbf{e}_u| \, \mathrm{d}\Omega + \int_\Gamma |(\mathbf{r}_\sigma^*)^\mathrm{T} \mathbf{e}_\mathrm{u}| \, \mathrm{d}\Gamma} \tag{1}$$

With the support of FP7 ITN Funding under Grant No. 289361 'INSIST', Ministerio de Economía y Competitividad of Spain (DPI2010-20542), FPU program (AP2008-01086) and Generalitat Valenciana (PROMETEO/2012/023).

where $\mathbf{e}_u := \mathbf{u}_u^* - \mathbf{u}^h$ is the difference between the kinematically admissible recovered solution and the FE solution and $\mathbf{s}_\sigma^*$ and $\mathbf{r}_\sigma^*$ refers to the lacks of internal and boundary equilibrium.

The numerical results presented on the left hand side of Figure 1 show the accuracy of the error estimator of the recovered solution. The black line is the error in energy norm of the FE solution, the red line is the exact error of the recovered solution and the blue line is the proposed error estimator for the error of the recovered solution. The error estimator of the recovered solution allows us to use a h-adaptive refinement strategy based on the use of the recovered solution as output the the FE analysis. The proposed strategy consist of the following steps: 1) Generate a FE mesh. 2) Solve the FE problem. 3) Evaluate the local error estimate of FE solution $(\mathbf{u}^h, \boldsymbol{\sigma}^h)$. 4) Evaluate the global value of the error estimate of the recovered solution $(\mathbf{u}_u^*, \boldsymbol{\sigma}_\sigma^*)$ (costless procedure). 5) If target error is smaller than the estimated error of the recovered solution continue to step 6, else stop the process. 6) Generate a h-adapted mesh using the local FE error estimation. 7) Go to step 2. The right hand side of Figure 1 shows the computational cost required for a given accuracy. The Figure clearly shows the time savings when the recovered solution is used in combination with the proposed h-adaptive strategy, making this new approach extremely interesting.

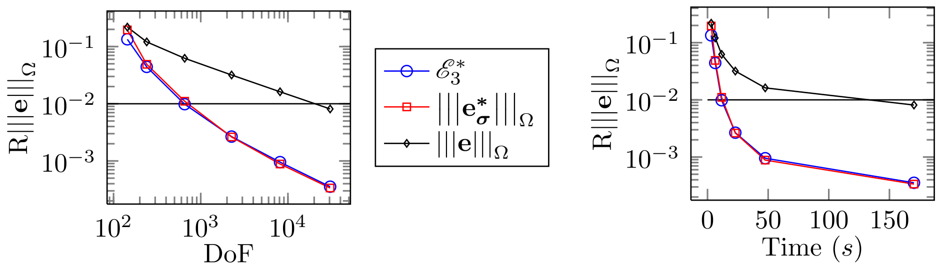

Figure 1: Pipe under internal pressure. h-adaptive analysis with Q4 elements. The black horizontal line represents the prescribed relative error in energy norm (1%).

REFERENCES

[1] I. Babushka, T. Strouboulisb and S.K. Gangarajb. A posteriori estimation of the error in the recovered derivatives the finite element solution. *Computational Methods*, Vol. **150**, 369–396, 1997.

[2] N.E.Wiberg and F. Abdulwahab. Error estimation with postprocessed finite element solutions. *Computers and structures*, Vol. **64**(1-4), 113–137, 1997.

[3] E. Nadal, J.J. Ródenas, J. Albelda, M. Tur, J.E. Tarancón, F.J. Fuenmayor. Efficient finite element methodology based on Cartesian Grids: Applicantion to structural shape optimization. *Abstract and applied analysis*, Pages 1–19, 2013.

[4] E. Nadal. Cartesian grid FEM (cgFEM): High performance h-adaptive FE analysis with efficient error contro. Application to structural shape optimization. PhD Thesis. *Universitat Politècnica de València*, 2014.