

# MONOTONE FINITE VOLUME SCHEME FOR MULTIPHASE FLOWS

Kirill Nikitin<sup>1</sup>, Kirill Terekhov<sup>2</sup> and Yuri Vassilevski<sup>3</sup>

Institute of Numerical Mathematics of the Russian Academy of Sciences,  
8 Gubkina St., Moscow, 119333, Russia

<sup>1</sup> nikitin.kira@gmail.com, <sup>2</sup> kirill.terekhov@gmail.com, <sup>3</sup> yuri.vassilevski@gmail.com

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We present a new nonlinear monotone finite volume method for diffusion equation and its application to two- and three-phase flow models. We consider full anisotropic discontinuous diffusion or permeability tensors on conformal polyhedral meshes.

The approximation of the diffusive flux uses the nonlinear two-point stencil which provides the conventional 7-point stencil for the discrete diffusion operator on cubic meshes. We show that the quality of the discrete flux in a reservoir simulator has great effect on the front behavior and the water breakthrough time. We compare conventional linear and new nonlinear two-point flux approximations, and linear multi-point flux approximation. The new nonlinear scheme has a number of important advantages over the traditional linear discretizations.

Compared to the linear two-point flux approximation, the nonlinear scheme demonstrates low sensitivity to grid distortions and provides appropriate approximation in case of full anisotropic permeability tensor. For non-orthogonal grids or full anisotropic permeability tensors the conventional linear scheme provides no approximation, while the nonlinear flux is still first-order accurate. The computational work for the new method is higher than the one for the conventional two-point flux approximation, yet it is rather competitive.

Compared to multi-point flux approximation, the new scheme provides sparser algebraic systems and thus is less computational expensive. Moreover, it is monotone which means that the discrete solution preserves the non-negativity of the differential solution.

## REFERENCES

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