11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

HIGHER-ORDER LEAST-SQUARES RECONSTRUCTION FOR TURBULENT AERODYNAMIC FLOWS

Alireza Jalali[1] and Carl Ollivier-Gooch[2]

[1] PhD candidate, Mechanical Engineering Department, University of British Columbia, Vancouver, Canada, arjalali@interchange.ubc.ca

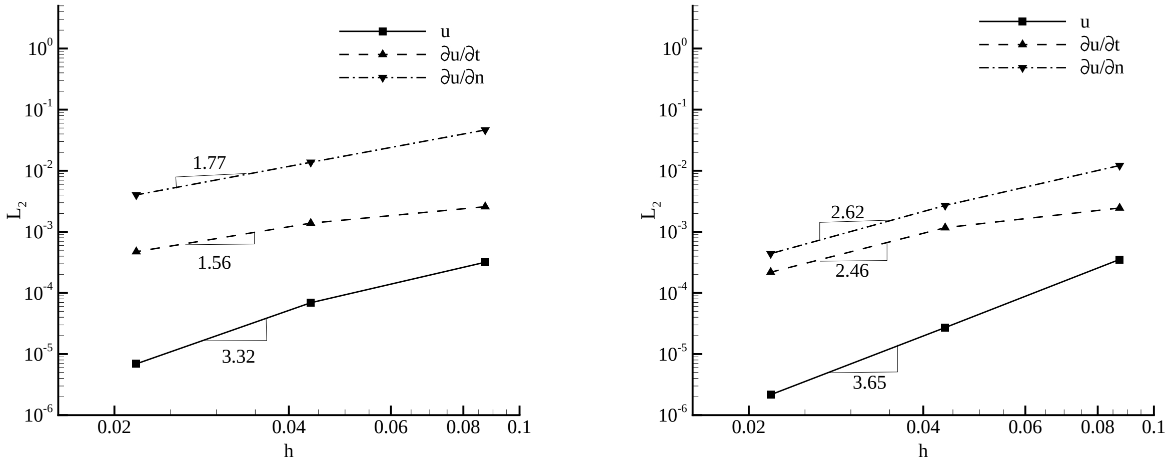
[2] Professor, Mechanical Engineering Department, University of British Columbia, Vancouver, Canada, cfog@mech.ubc.ca

Key words: *Finite volume, Anisotropic mesh, Reconstruction, Turbulent flows.*

Higher order – higher than second order – computation of fluid dynamic equations on unstructured meshes is motivated by the potentially significant cost reduction and the flexibility of unstructured meshes for complex geometries. The key point in the development of higher-order methods for unstructured meshes is a higher-order polynomial reconstruction of primitive variables within the control volumes based on nearby control volume average data. Least-squares k-exact reconstruction [1] is one candidate for this reconstruction and the one we focus on in this work.

In the context of turbulent flows for aerodynamic applications, it is quite common to have cells whose aspect ratios are $10,000$ or higher. Furthermore, aerodynamic configurations typically consist of curved surfaces on which the flow boundary layer develops. As a result, the simulation of high Reynolds number turbulent flows requires sufficiently accurate polynomial approximation on high aspect ratio meshes with finite curvature. Mavriplis [2] examined the accuracy of the least-squares technique for second-order discretization on unstructured meshes in the presence of surface curvature. For a cell-centered method on triangular meshes with high curvature, the computed cell gradient exhibits poor accuracy even after weighting the least-squares system by geometrically close data.

In this paper, we will introduce a new baseline for higher-order solution reconstruction on high aspect ratio curved meshes. Since the linear representation of curved boundaries is not adequate for higher-order solutions, the boundaries must be curved as well. We propagate the mesh deformation into the domain interior to prevent faces from intersecting near curved boundaries using a linear elasticity analogy. For solution reconstruction, a curvilinear local mapping from the physical space into a tangential-normal coordinate system is used for cells with high curvature near the walls. The mapping is obtained by the distance function and constructed tangential direction at the cells' reference points. An auxiliary least-squares system is solved to give the coefficients of quadratic and cubic



(a) Third-order (b) Fourth-order

Figure 1: Reconstruction error (relative error for normal derivatives)

mappings for third- and fourth-order solutions, respectively. Far from the wall where the cells' curvature is reduced as in wake region, the traditional reconstruction can be applied using the coordinate system aligned with the principle axes of the control volumes.

As a preliminary test on curved cells with high aspect ratio near the wall, we investigate the accuracy of higher-order reconstructed solution and derivatives. Figure 1 shows the L_2-norm of error on a sequence of triangular unstructured meshes over a 40 degree arc where the maximum aspect ratio is about $7,000$ and h is the mesh spacing in the circumferential direction. The turbulent boundary layer profile of Reichardt [3] is used as the test function where y^+ is computed based on the distance from the wall and the friction velocity, u^*, is obtained by the one-seventh power law at $Re_L = 10^7$. Our result shows that the error associated with the solution reconstruction on highly anisotropic cells with high curvature is reasonably small and within the expected order of accuracy. In the final paper, we will present the details of reconstruction paradigm along with accuracy analysis results on more general meshes typically used in computational aerodynamics.

REFERENCES

[1] Timothy J. Barth. Recent developments in high order k-exact reconstruction on unstructured meshes. AIAA paper 93-0668, January 1993.

[2] D. J. Mavriplis. Revisiting the least-squares procedure for gradient reconstruction on unstructured meshes. In *Proceedings of the Sixteenth AIAA Computational Fluid Dynamics Conference*, 2003.

[3] H. Reichardt. Vollstndige darstellung der turbulenten geschwindigkeitsverteilung in glatten leitungen. *ZAMM - Journal of Applied Mathematics and Mechanics*, 31:208–219, 1951.