11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

BOUNDARY ELEMENT METHODS WITH A H_{DIV} SCALAR PRODUCT FOR ELECTROMAGNETIC WAVE SCATTERING PROBLEMS

Kazuki Niino[1] and Naoshi Nishimura[2]

[1] Kyoto University, Yoshidahonmachi Sakyo-ku Kyoto, Japan, niino@acs.i.kyoto-u.ac.jp
[2] Kyoto University, Yoshidahonmachi Sakyo-ku Kyoto, Japan, nchml@i.kyoto-u.ac.jp

Key words: *Electromagnetic wave scattering problems,Low frequency problems*

1 introduction

Boundary element methods (BEMs) are efficient numerical methods for electromagnetic wave scattering problems. However, it is known that BEMs show bad accuracy for electromagnetic problems at low frequencies. This problem is called "low-frequency problem" and various methods which remedy this problem were suggested[1, 2]. In this paper, we will propose a new method which accurately solves electromagnetic wave scattering problems at low frequencies by utilizing a H_{div} scalar product for discretisation.

2 formulation

We consider a scatterer with a simple domain $\Omega^- \in \mathbb{R}^3$ enclosed by a smooth boundary Γ. Ω^+ is denoted by $\mathbb{R}^3 \backslash \overline{\Omega^-}$. We are interested in solving the following boundary value problems:

$$\begin{aligned}
&\nabla \times \boldsymbol{E} = \mathrm{i}\omega\mu^{\pm}\boldsymbol{H}, \quad \nabla \times \boldsymbol{H} = -\mathrm{i}\omega\varepsilon^{\pm}\boldsymbol{E} \quad \text{in}\, \Omega^{\pm} \\
&\boldsymbol{m} := \boldsymbol{E}^{+} \times \boldsymbol{n} = \boldsymbol{E}^{-} \times \boldsymbol{n}, \quad \boldsymbol{j} := \boldsymbol{n} \times \boldsymbol{H}^{+} = \boldsymbol{n} \times \boldsymbol{H}^{-} \quad \text{on}\, \Gamma
\end{aligned}$$

subjecting to the raditation conditions for the scattering fields ($\boldsymbol{E}^{\mathrm{sca}}$ and $\boldsymbol{H}^{\mathrm{sca}}$), where $\boldsymbol{E}$ and $\boldsymbol{H}$ are unknown electric and magnetic fields, ω is the frequency, $\varepsilon^{\pm}$ and $\mu^{\pm}$ are the permittivity and permeability of $\Omega^{\pm}$ and the scatterd fields are defined by $(\boldsymbol{E}^{\mathrm{sca}}, \boldsymbol{H}^{\mathrm{sca}}) = (\boldsymbol{E} - \boldsymbol{E}^{\mathrm{inc}}, \boldsymbol{H} - \boldsymbol{H}^{\mathrm{inc}})$ with the incident waves denoted by $\boldsymbol{E}^{\mathrm{inc}}$ and $\boldsymbol{H}^{\mathrm{inc}}$ in the exterior domain Ω^+, respectively.

To solve this problem, we will use a BEM with the Poggio-Miller-Chang-Harrington-Wu-

Tsai (PMCHWT) formulation:

$$(\Psi^+_{ij} + \Psi^-_{ij})m_j - \mathrm{i}\omega(\mu^+\Phi^+_{ij} + \mu^-\Phi^-_{ij})j_j = E^{\mathrm{inc}}_i, \quad (1)$$

$$\mathrm{i}\omega(\varepsilon^+\Phi^+_{ij} + \varepsilon^-\Phi^-_{ij})m_j + (\Psi^+_{ij} + \Psi^-_{ij})j_j = -H^{\mathrm{inc}}_i, \quad (2)$$

where

$$\Psi_{kl} = \mathrm{e}_{kjl}\partial_j G(x), \quad \Phi_{kl} = \left(\delta_{kl} + \frac{1}{k^2}\partial_k\partial_l\right) G(x).$$

Note that we use the summation convention to repeated indices in these formulae.

3 discretisation

We will utilise the H_{div} scalar product

$$(\boldsymbol{u}, \boldsymbol{v})_{H_{\mathrm{div}}(\Gamma)} := (\boldsymbol{u}, \boldsymbol{v})_{L^2_T(\Gamma)} + c(\mathrm{div}_S\boldsymbol{u}, \mathrm{div}_S\boldsymbol{v})_{L^2(\Gamma)}$$

for discretising the boundary integral equations in (1) and (2), where c is a positive constant. Discretised boundary integral equations with this scalar product are obtained as follows:

$$\begin{aligned}&(\boldsymbol{s}_i, \boldsymbol{n} \times \{(\boldsymbol{\Psi}^+ + \boldsymbol{\Psi}^-)\boldsymbol{m} - \mathrm{i}\omega(\mu^+\boldsymbol{\Phi}^+ + \mu^-\boldsymbol{\Phi}^-)\boldsymbol{j}\})_{L^2_T(\Gamma)} + c(\mathrm{div}_S\boldsymbol{s}_i, \boldsymbol{n} \cdot \{\mathrm{i}\omega(\mu^+\boldsymbol{\Psi}^+ + \mu^-\boldsymbol{\Psi}^-)\boldsymbol{j} \\ &- ({k^+}^2\boldsymbol{\Phi}^+ + {k^-}^2\boldsymbol{\Phi}^-)\boldsymbol{m}\})_{L^2(\Gamma)} = -(\boldsymbol{s}_i, \boldsymbol{E}^{\mathrm{inc}} \times \boldsymbol{n})_{L^2_T(\Gamma)} - \mathrm{i}\omega\mu^+ c(\mathrm{div}_S\boldsymbol{s}_i, \boldsymbol{n} \cdot \boldsymbol{H}^{\mathrm{inc}})_{L^2(\Gamma)}\end{aligned} \quad (3)$$

$$\begin{aligned}&(\boldsymbol{s}_i, \boldsymbol{n} \times \{\mathrm{i}\omega(\varepsilon^+\boldsymbol{\Phi}^+ + \varepsilon^-\boldsymbol{\Phi}^-)\boldsymbol{m} + (\boldsymbol{\Psi}^+ + \boldsymbol{\Psi}^-)\boldsymbol{j}\})_{L^2_T(\Gamma)} + c(\mathrm{div}_S\boldsymbol{s}_i, \boldsymbol{n} \cdot \{-({k^+}^2\boldsymbol{\Phi}^+ + {k^-}^2\boldsymbol{\Phi}^-)\boldsymbol{j} \\ &- \mathrm{i}\omega(\varepsilon^+\boldsymbol{\Psi}^+ + \varepsilon^-\boldsymbol{\Psi}^-)\boldsymbol{m}\})_{L^2(\Gamma)} = -(\boldsymbol{s}_i, \boldsymbol{n} \times \boldsymbol{H}^{\mathrm{inc}})_{L^2_T(\Gamma)} - \mathrm{i}\omega\varepsilon^+ c(\mathrm{div}_S\boldsymbol{s}_i, \boldsymbol{n} \cdot \boldsymbol{E}^{\mathrm{inc}})_{L^2(\Gamma)}\end{aligned} \quad (4)$$

where $\boldsymbol{s}_i$ is a testing function. Note that we have to use Buffa-Christiansen basis function for $\boldsymbol{s}_i$ if we expand the unknown functions $\boldsymbol{m}$ and $\boldsymbol{j}$ with the RWG basis function since, compared with the boundary integral equatoins in (1) and (2), the second elements of the first terms in the left-hand sides of equations (3) and (4) have the term $\boldsymbol{n}\times$. Discretised integral equations in (3) and (4) naturally includes both tangential and normal components of equations (1) and (2), thus, a solution obtained by solving these equations is expected to be more accurate. We set the constant c as $1/\omega$ so that the normal and tangential components of equations (1) and (2) have similar amounts for small frequency ω.

REFERENCES

[1] Francesco Andriulli, Kristof Cools, Ignace Bogaert, and Eric Michielssen. *IEEE Transactions on Antennas and Propagation*, Vol. 61, No. 4, pp. 2077–2087, 2013.

[2] Zhi Guo Qian and Weng Cho Chew. *Microwave and Optical Technology Letters*, Vol. 50, No. 10, pp. 2658–2662, 2008.