11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

A NURBS BASED COLLOCATION APPROACH FOR SB-FEM

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Key words: *Scaled boundary finite element method (SB-FEM), Collocation method; NURBS basis functions, Exact integration of the boundary, Trimmed NURBS.*

The paper is concerned with a new numerical method to solve the in-plane motion problem of elastic solids. A novel element formulation is derived, which is based on the so-called scaled boundary finite-element method (SB-FEM). The method was introduced by Wolf and Song [1]. It is a specific type of finite element, where only the boundary of each element is discretized. The domain inside the element is described by a scaling factor. The element domain is parameterized with the radial scaling factor and a parameter in circumferential direction. Applying the weak form in circumferential direction the governing partial differential equations of elasticity are transformed to the scaled boundary finite element equation. Finally, an ordinary differential equation of Euler type for the displacements is observed, where the displacements are a function of the radial scaling parameter.

The common solution procedure for the scaled boundary finite element equation is based on the eigenvalue method, see e.g. [2]. The displacement field is formulated as a power series in terms of the radial scaling parameter. The eigenvalues determine the powers of the terms in the series. The corresponding eigenvectors describe the angular variations. However, present eigenvalue method for solving the SB-FEM requires additional treatments for multiple eigenvalues with parallel eigenvectors, which results in logarithmic terms for the solutions. It is remarked that the accuracy of the eigenvalue method deteriorates when the solution approaches one within the logarithmic functions, see [3].

The scope of this work is to develop a novel numerical scheme to solve the scaled boundary finite-element equation, which is guided by the desire to accomplish the following objectives:

i) Efficiency and accuracy comparable to those of the finite element method.

ii) Numerical stability for finite elements with extreme shape, such as distorted elements.

iii) Computation of domains described by surfaces in combination with boundary curves, such as trimmed NURBS.

iv) Possibility to combine FEM and SB-FEM in one domain according to the property of the mesh.

In the proposed implementation, the isogeometric collocation method will be employed to solve the scaled boundary finite-element equation. Isogeometric collocation is a method introduced in [4] to solve differential equations. The higher continuity provided by NURBS allows to use collocation of the strong form of the equation instead of using the Galerkin method for the weak form. It is more efficient than Galerkin based schemes [5].

A finite-dimensional space of candidate solutions (NURBS basis functions) and a number of points in the domain - called collocation points - are constructed. Collocation is applied between the center and the boundary of the element. Thus, only one-dimensional collocation is used. No two-dimensional collocation or integration over the surface is necessary. The boundary is integrated exactly without additional numerical effort.

The given equation is evaluated at all collocation points, assembled to a system of linear algebraic equations and solved. If a certain set of collocation points is used, the method is numerically stable [4]. Further applications of this approach can be extended to trimmed-NURBS, because the boundary-oriented character of SB-FEM ideally corresponds to the trimming of elements with NURBS curves, which is the way trimmed-NURBS surfaces are described.

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