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ON THE VERIFICATION OF PGD REDUCED-ORDER MODELS

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In current computational mechanics practice, multidimensional as well as multiscale or parametric models encountered in a wide variety of scientific and engineering fields often require either the resolution of significantly large complexity problems or the direct calculation of very numerous solutions of such complex models. In this framework, the use of model order reduction allows to dramatically reduce the computational requirements engendered by the increasing model complexity. Over the last few years, model order reduction techniques have sparked a growing interest in the whole scientific community [1, 2]. These appealing methods are based on separated variables representations of the solution of multi-parameter models lying in tensor product spaces. They enable to circumvent the terrific curse of dimensionality as the associated solution complexity scales linearly with the dimension of the tensor product space, whereas classical brute force approaches, such as mesh-based (or grid-based) approximation methods, face an exponentially growing solution complexity. Proper Generalized Decomposition (PGD) is currently one of the most popular Reduced-Order Modeling (ROM) techniques which can be interpreted as an extension of Proper Orthogonal Decomposition (POD). It allows the a priori construction of separated variables representations of the model solution without requiring any knowledge or information about this one (contrary to POD). Low-dimensional PGD reduced basis functions (or modes) are first constructed online on the fly by sequentially solving a series of few tractable simple problems. The resulting PGD-based approximate solution can then be computed offline, as it is defined explicitly in terms of all model parameters. Despite the good performances of PGD techniques, a major issue concerns the control of PGD reduced-order models and the development of robust and efficient verification tools able to assess the quality of PGD-based numerical approximations.

A few works have been devoted to the development of a posteriori error estimation methods allowing to control and assess the numerical quality of PGD reduced-order models. Pioneering works provided goal-oriented error indicators designed for adaptivity purposes without furnishing reliable and strict error bounds [3]. Subsequently, guaranteed and robust error estimators have been recently introduced in [4, 5, 6] in order to control the precision of PGD reduced-order approximations for multi-parameter linear elliptic and parabolic problems depending on a moderate number of parameters. The underlying verification procedure relies on the concept of Constitutive Relation Error (CRE) [7] along with the construction of associated admissible fields. It enables to capture various error sources (space and time discretization errors, truncation error in the finite sums decomposition, etc.) and to assess their relative contributions by means of appropriate error indicators in order to drive adaptive strategies for the optimal construction of PGD reduced-order approximations. In this work, we propose an extension of this PGD-verification method to the case of large sets of model parameters. The progressive Galerkin-based PGD technique is used to build an approximate separated representation of the model solution via a greedy algorithm. Numerical experiments carried out on linear elasticity problems with numerous fluctuating model parameters illustrate the behavior and the capabilities of the verification method in the PGD framework.

REFERENCES

[1] F. Chinesta, A. Ammar and E. Cueto. Recent Advances and New Challenges in the Use of the Proper Generalized Decomposition for Solving Multidimensional Models. *Archives of Computational Methods in Engineering*, Vol. **17(4)**, 327–350, 2010.

[2] A. Nouy. A priori model reduction through Proper Generalized Decomposition for solving time-dependent partial differential equations. *Computer Methods in Applied Mechanics and Engineering*, Vol. **199(23-24)**, 1603–1626, 2010.

[3] A. Ammar, F. Chinesta, P. Díez and A. Huerta. An error estimator for separated representations of highly multidimensional models. *Computer Methods in Applied Mechanics and Engineering*, Vol. **199(25-28)**, 1872–1880, 2010.

[4] P. Ladevèze and L. Chamoin. On the verification of model reduction methods based on the proper generalized decomposition. *Computer Methods in Applied Mechanics and Engineering*, Vol. **200(23-24)**, 2032–2047, 2011.

[5] L. Chamoin and P. Ladevèze. Robust control of PGD-based numerical simulations. *European Journal of Computational Mechanics/Revue Européenne de Mécanique Numérique*, Vol. **21(3-6)**, 195–207, 2012.

[6] P. Ladevèze and L. Chamoin. *Toward guaranteed PGD-reduced models*. Bytes and Science. CIMNE: Barcelona, 2013.

[7] P. Ladevèze and J. P. Pelle. *Mastering Calculations in Linear and Nonlinear Mechanics*. Springer, New York, 2004.