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A COMPARISON OF THREE DISCRETIZATION METHODS FOR THE SIMULATION OF HIGHLY ANISOTROPIC RESERVOIRS WITH TETRAHEDRAL GRIDS

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The performance of three methods applied to the discretization of reservoir flow models in tetrahedral grids is compared in this work. The methods considered follow different approaches in order to approximate the differential equations that model flows inside reservoirs. The first one is a cell-center approach where mass balances are carried out considering grid cells as control volumes and approximating interface fluxes with the well-known MPFA-O method [1]. The second method considered is the element-based finite volume method (EbFVM), mostly known in the literature as control-volume finite element method (CVFEM) [2] . This method follows a cell-vertex approach. Finally, the third is the mimetic finite difference (MFD) method [3], in which differential operators are discretized in a way that they satisfy fundamental identities. By construction, those three methods are able to deal with full-tensor permeabilities.

Since our main interest is the application of some of those methods in highly anisotropic problems in complex geometries discretized with tetrahedral grids, we carried out a series of numerical experiments in order to determine the more convenient option. In one of those experiments, we solved a single-phase flow problem with known analytical solution in a cubic domain with homogeneous properties. A series of progressively refined tetrahedral grids were used in order to estimate the convergence rate of pressure. Some of the results obtained with the three methods compared are shown in Fig.1. The first three graphs show the decrease of the pressure error norm as cell size is reduced, for four different anisotropy ratios. All methods show near second-order convergence, although with some significant differences. In the case of MPFA, it was not possible to obtain numerical solutions for anisotropy ratios greater than 10. In those cases, linear systems of equations with large negative coefficients arose and we could not solve them with any of several solution methods. The drawback with the results obtained with MFD is that the error level increases excessively as the anisotropy ratio grows. On the other hand, EbFVM is the method which shows the most reasonable behavior. Although the error level with EbFVM grows also with the increasing of the anisotropy ratio, the maximum values are far below of those of MFD. Another aspect worth of considering is the size of the system of equations that arises with each method. That is compared in Fig.1(d), for the three most refined grids in the series. Pressure unknowns in the linear system of MFD are related to faces, in MPFA to cell centers, and in EbFVM to vertices. Because of that, the number of unknowns in EbFVM is an order of magnitude lower that in MFD, for the same grid. For the specific application considered, EbFVM seems to be the best choice.

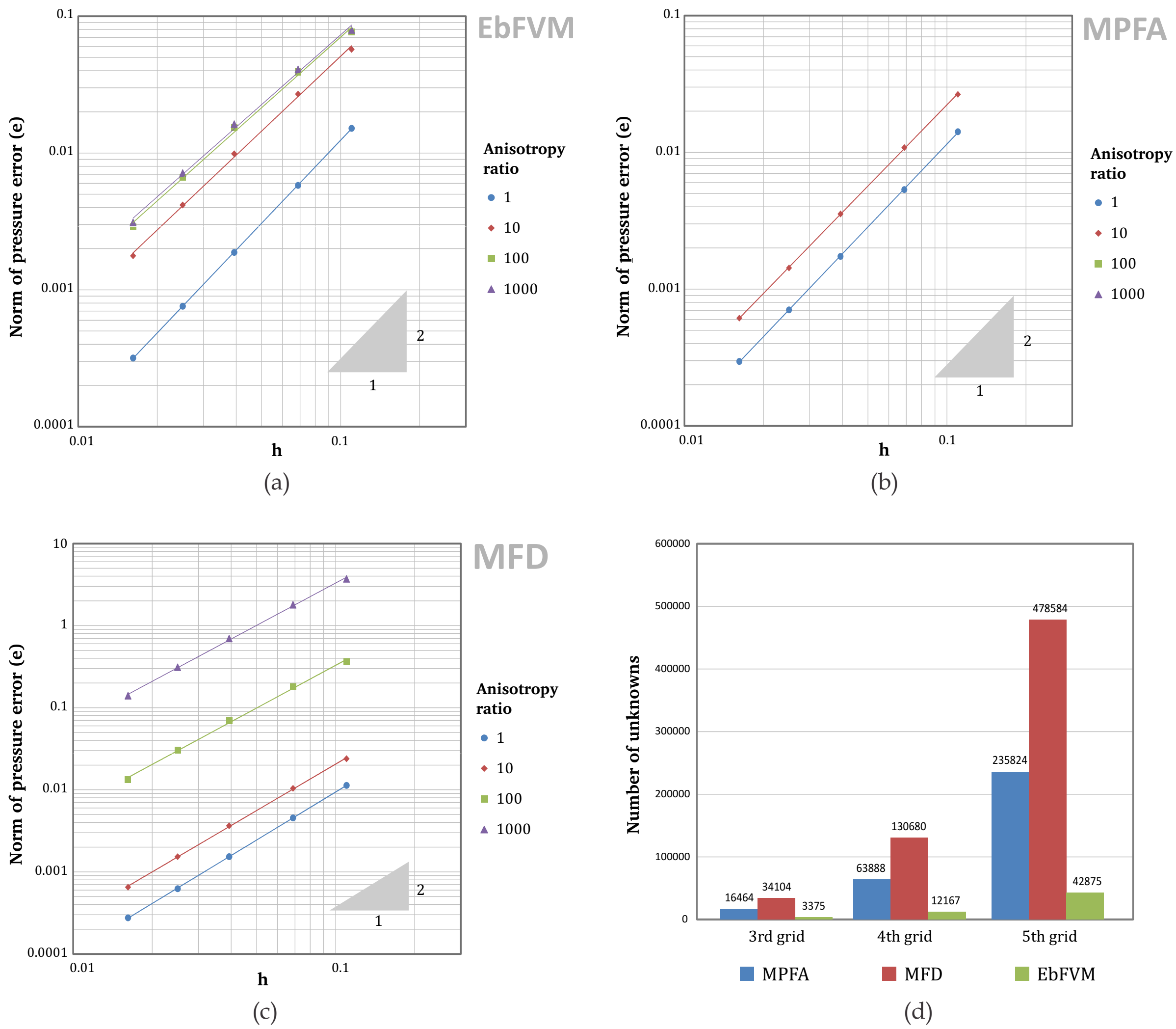

Figure 1: Convergence of pressure error on (a) EbFVM, (b) MPFA and (c) MFD. (d) Comparison of the number of unknowns solved in each method.

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