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A nonlinear finite element for simulation of dynamics of beam structures using multibody system approach

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Many engineering dynamic problems can be simulated as beam structures. For example, such models are applied in well drilling. Dynamic simulation allows optimizing shape of well bore and operations parameters of the drilling. Calculation speed of simulation of dynamics of a drill string depends on size of matrices of a model and on effectiveness of numerical methods.

The approach to simulation of dynamics of drill strings are suggested by the authors in [1]. The drill string is presented as a set of uniform beams connected via force elements. Flexibility of the beams is simulated using the modal approach. Thus, each beam has at least twelve degrees of freedom: six coordinates define position and orientation of a local frame and six modes are used for modeling flexibility. For simulation of the drilling processes, implicit Park method with Jacobian of stiff forces is used [2]. Analysis of vibration, rock cutting, buckling and post-buckling behaviour and other processes of the drilling can be successfully modeled using the approach. But it has some disadvantages.

Firstly, number of degrees of freedom can be decreased if a single nonlinear finite element model is used instead of the model including great number of beam subsystems. Secondly, simulation of real rotation of the drill string using the modal approach related to the problem with calculation of Jacobians from stiff force elements. The expressions of the modal coordinate derivatives of the stiff forces are variable since each mode is calculated in the local frame of the beam and rotate together with the frame.

To increase simulation effectiveness, geometrically nonlinear beam finite element is developed (Figure 1). The vector of generalized coordinates of the element has the following form:

$$\mathbf{q} = \{\mathbf{r}_1^{\mathrm{T}} \quad \boldsymbol{\varphi}_1^{\mathrm{T}} \quad \mathbf{r}_2^{\mathrm{T}} \quad \boldsymbol{\varphi}_2^{\mathrm{T}}\}^{\mathrm{T}}, \tag{1}$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the absolute position vectors of the two nodes, while $\boldsymbol{\varphi}_1$ and $\boldsymbol{\varphi}_2$ are the vectors of the orientation angles (Cardan angles are used here: $\boldsymbol{\varphi}_i = \{\alpha_i \quad \beta_i \quad \gamma_i\}^{\mathrm{T}}$) for the nodal cross sections. Also, the dependent local coordinates $\mathbf{u}$ are introduced to represent small displacements of the right-hand-side cross section relative to the left-hand-side one:

$$\mathbf{u}(\mathbf{q}) = \begin{Bmatrix} \{u_1 \ u_2 \ u_3\}^{\mathrm{T}} \\ \{\alpha_1 \ \alpha_2 \ \alpha_3\}^{\mathrm{T}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{A}_1^{\mathrm{T}}\{\mathbf{r}_2 - \mathbf{r}_1\} - \{L \ \ 0 \ \ 0\}^{\mathrm{T}} \\ \boldsymbol{\alpha}(\mathbf{A}_1^{\mathrm{T}}\mathbf{A}_2) \end{Bmatrix}, \tag{2}$$

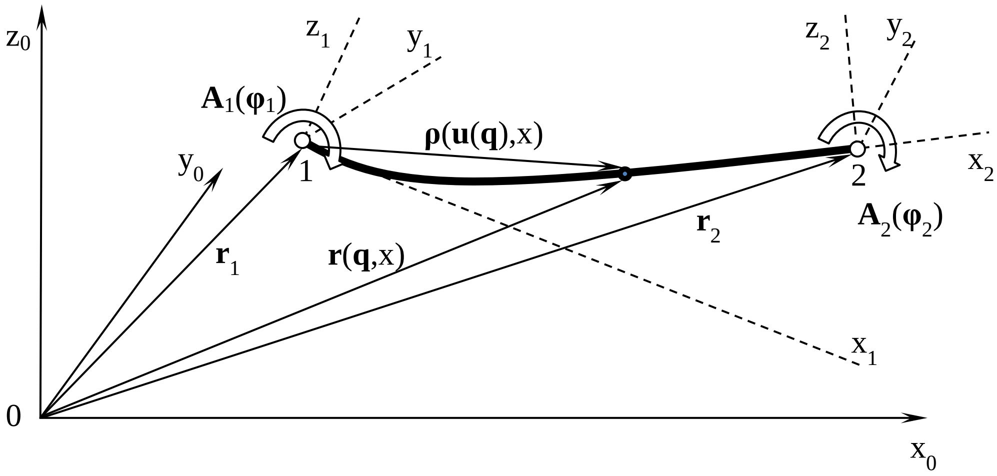

Figure 1. Geometry and generalized coordinates of a single beam element.

Here, A_1 and A_2 are matrices of orientation of the nodal cross-sections, and $\boldsymbol{\alpha}(\mathbf{A})$ is the vector-valued function of a matrix argument, which returns the Cardan angles, given the orientation matrix A; $\alpha_1=(A_{23}-A_{32})/2$, *etc*. This allows introducing local displacement field $\boldsymbol{\rho}(\mathbf{u}(\mathbf{q}),x)$ of the beam by employing local shape function matrix $\mathbf{N}(x)$:

$$\boldsymbol{\rho}(\mathbf{u}(\mathbf{q}),x)=\mathbf{N}(x)\,\mathbf{u}(\mathbf{q})+\{x\ \ 0\ \ 0\}^{\mathrm{T}},\ \mathbf{N}(x)=\begin{vmatrix} \xi & 0 & 0 & 0 & 0 & 0 \\ 0 & 3\xi^2-2\xi^3 & 0 & 0 & 0 & L(\xi^3-\xi^2) \\ 0 & 0 & 3\xi^2-2\xi^3 & 0 & -L(\xi^3-\xi^2) & 0 \end{vmatrix},\ \xi=\frac{x}{L}. \tag{3}$$

The global position of an arbitrary point can be computed as follows: $\mathbf{r}(\mathbf{q},x)=\mathbf{r}_1+\mathbf{A}_1\,\boldsymbol{\rho}(\mathbf{q},x)$, which allows computing kinetic and potential energy and obtaining the equations of motion:

$$\mathbf{M}(\mathbf{q})\,\ddot{\mathbf{q}}+\mathbf{k}(\dot{\mathbf{q}},\mathbf{q})+\mathbf{f}(\mathbf{q})=\mathbf{0}$$

where

$$\mathbf{M}(\mathbf{q})=\int_0^L \mu\,\mathbf{S}^{\mathrm{T}}\mathbf{S}\,\mathrm{d}x,\ \ \mathbf{k}(\dot{\mathbf{q}},\mathbf{q})=\int_0^L \mu\,\mathbf{S}^{\mathrm{T}}\dot{\mathbf{S}}\,\mathrm{d}x\,\dot{\mathbf{q}},\qquad \mathbf{f}(\mathbf{q})=\mathbf{U}(\mathbf{q})^{\mathrm{T}}\mathbf{K}\,\mathbf{u}(\mathbf{q}),$$

S and **U** are the Jacobian matrices of the following form:

$$\mathbf{S}(\mathbf{q},x)=\frac{\partial\mathbf{r}}{\partial\mathbf{q}}=\begin{bmatrix}\mathbf{I} & -\mathbf{A}_1\tilde{\boldsymbol{\rho}}(\mathbf{q},x)\mathbf{A}_1^{\mathrm{T}}\mathbf{B}_1 & \mathbf{O} & \mathbf{O}\end{bmatrix}+\mathbf{A}_1\mathbf{N}(x)\,\mathbf{U}(\mathbf{q}), \tag{4}$$

$$\mathbf{U}(\mathbf{q})=\frac{\partial\mathbf{u}}{\partial\mathbf{q}}=\begin{bmatrix}-\mathbf{A}_1^{\mathrm{T}} & \mathbf{A}_1^{\mathrm{T}}\widetilde{\{\mathbf{r}_2-\mathbf{r}_1\}}\mathbf{B}_1 & \mathbf{A}_1^{\mathrm{T}} & \mathbf{O} \\ \mathbf{O} & -\mathbf{A}_1^{\mathrm{T}}\mathbf{B}_1 & \mathbf{O} & \mathbf{A}_1^{\mathrm{T}}\mathbf{B}_2\end{bmatrix},\ \mathbf{B}_i=\frac{\partial\boldsymbol{\omega}_i}{\partial\dot{\boldsymbol{\varphi}}_i}=\frac{\partial\{*\tilde{\boldsymbol{\omega}}_i\}}{\partial\dot{\boldsymbol{\varphi}}_i}=\frac{\partial\{*[\dot{\mathbf{A}}_i\mathbf{A}_i^{\mathrm{T}}]\}}{\partial\dot{\boldsymbol{\varphi}}_i}$$

The models created using the developed finite element were verified on the tasks having analytical solution. Calculation speed under simulation of dynamics of long drill strings is increased approximately two times as compared with the modal approach. The results obtained applying both approaches are discussed in the report.

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[1] D. Pogorelov, G. Mikheev, N. Lysikov, L. Ring, R. Gandikota and N. Abedrabbo. A multibody system approach to drill string dynamics modeling, *2012 Proc. of the ASME 11th Biennial Conf. on Eng. Systems Design and Analysis (ESDA2012)*, Vol. 4, 53-62, 2012.

[2] D. Yu. Pogorelov. Jacobian matrices of the motion equations of a system of bodies. *Journal of Computer and Systems Sciences International* 46(4)#7, 563–577, 2007.