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A TWO STEP PROCESS FOR SHAPE OPTIMIZATION IN COMPUTATIONAL FLUID DYNAMICS

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Introduction

This work concerns a shape optimization problem, i.e. that of minimizing an objective function $J(\Omega)$ of the domain variable Ω, in the context of computational fluid dynamics (CFD). We investigate a twofold approach, consisting in a preliminary use of the so-called SIMP (Solid Isotropic Material with Penalization) method, followed by an optimization method relying on the shape derivative. The numerical scheme has been implemented using `FreeFem++`[4] and involves a mesh adaptation stage to improve the boundary approximation as well as the numerical accuracy and efficiency of the method.

1 First stage: topology optimization

The first step is based on a classical SIMP formulation with a material distribution model proposed by Khadra [3] and used first by Borrvall [2] in fluid optimization. A fictitious solid domain is mimicked by using a Brinkmann penalization of the Stokes equation, a heuristic based on the theory of porous media: a term αu_i is added to the Stokes equation posed in each subdomain Ω_i of the fixed computational domain Ω, which then reads:

$$-\mu\Delta u_i + \alpha(\rho)u_i = f_i - \nabla p\,. \tag{1}$$

The density function ρ is defined over the entire domain Ω, and takes the value $\rho = 1$ (resp. $\rho = 0$) on the fluid (resp. solid) part. The inverse permeability α is defined as a function of the density ρ, and accounts for a penalization parameter.

In order to reduce the “losses” of the Stokes system, the optimization function $\Phi(\vec{u}, \rho)$ is devised to minimize the power dissipation of the fluid. The Stokes problem (1) is endowed with classical Dirichlet boundary conditions. This yields the optimization problem:
Find (u, p) *solution to Problem* (1) *such that:*

$$\min_{\rho} \Phi(u_i(\rho), \rho) = \int_{\Omega} \left(\mu\nabla u_i : e(u_i) + \alpha(\rho)||u_i||^2\right) \mathrm{d}x,$$

to which a volume constraint is added.

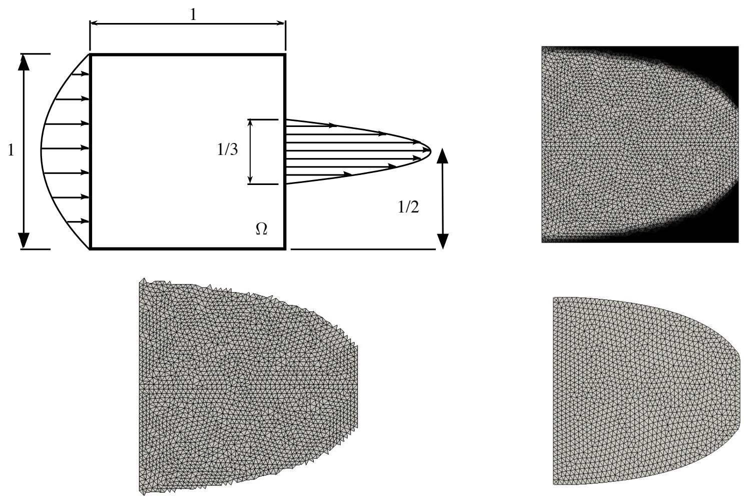

Figure 1: Optimization process with a maximum volume prescription of 0.85: *Upper left:* Initial shape and boundary conditions. *Upper right:* Density result of SIMP optimization. Lower *left:* Resulting domain corresponding to the $\rho = 0.5$ density isovalue. *Lower right:* final shape.

2 Second stage: shape optimization

The fluid domain boundary resulting from the previous stage is usually not very accurate (as it corresponds to an average isovalue of a density function). Hence, a 'geometric' shape optimization procedure based on an objective function $J(\Omega^0)$ of the fluid part Ω^0 of the domain is carried out on an unstructured (adapted) mesh in order to improve its description (see e.g. [1], Chap. 6): the analysis of the shape derivative of J makes it possible to compute a descent direction for J from a given shape Ω^0, as a vector field V_{Ω^0}.

3 Results

The diffuser example (see [2]) is implemented to validate our approach (Fig. 1). The SIMP optimization process yields a resulting shape that is then further optimized using the shape derivative method. At completion, a smooth explicit boundary is obtained for the optimal design with respect to the objective function.

REFERENCES

[1] G. Allaire, *Conception optimale de structures*, Mathématiques et Applications 58, Springer, Heidelberg (2006).

[2] T. Borrvall and J. Petersson, "Topology optimization of fluids in stokes flow," *International Journal for Numerical Methods in Engineering*, vol. 41, pp. 77–107, 2003.

[3] K. Khadra, P. Angot, S. Parneix, and J.-P. Caltagirone, "Fictitious domain approach for numerical modelling of navierstokes equations," *International Journal for Numerical Methods in Fluids*, vol. 34, no. 8, pp. 651–684, 2000.

[4] O. Pironneau, F. Hecht, A. Le Hyaric, *FreeFem++ version 2.15-1*, `http://www.freefem.org/ff++/`.