

## ABSORBING BOUNDARY CONDITIONS FOR TILTED TRANSVERSE ISOTROPIC ELASTIC MEDIA

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**Key words:** *Absorbing Boundary Conditions, Tilted Transverse Isotropy, Elastic Media*

The numerical simulation of wave propagation requires the design of Boundary Conditions (BC) that are able to attenuate artificial reflected waves generated by the boundaries of the computational domain. Besides, when considering Earth, the subsurface layers imply Tilted Transverse Isotropic (TTI) media. This anisotropic feature is essential for the accuracy of geophysics simulation, see for example [3].

Transverse Isotropy (TI) implies that physical properties are symmetric about an axis that is normal to a plane in which the physical properties are the same in all directions. When the axis is the vertical, it is called Vertical Transverse Isotropy (VTI), otherwise, the axis is described by angles (one in 2D, two in 3D) and it is called Tilted Transverse Isotropy (TTI). Fig. 1 illustrates 2D wavefronts in these different materials.

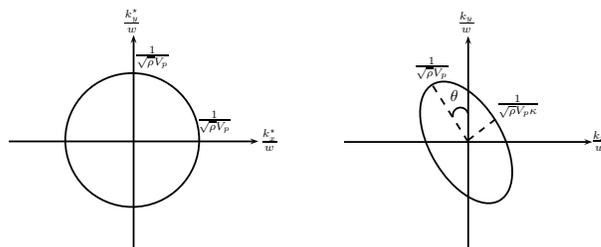


**Figure 1:** 2D wavefronts for isotropic (left), VTI (center) and TTI (right) media

A rigorous methodology for the construction of ABCs is based on the diagonalization of the elastodynamics system. This approach has been proposed by Enquist and Majda [2] for strongly hyperbolic systems. Nevertheless, in practice, it can quickly become uneasy to use because of coupling terms between P-waves and S-waves. A possible approach consists then in uncoupling these waves and constructing ABCs for each of them. This allows the construction of ABCs for VTI media but, unfortunately, this technique is not enough considering TTI. This is due to the characteristic angles of rotation that prevent from obtaining a local ABC.

We propose a new low-order ABC for TTI media obtained by the study of the geometry profile of the slowness curves of the waves. This idea of looking at the slowness curves is common when it concerns anisotropic waves, see for example Bécache et al. in [1] for 2D ABCs in anisotropic acoustic media; or the set of Savadatti and Guddati's articles, see in [4] and their references therein.

The method consists first in writing the parametrization of the slowness curves for P-waves and S-waves. The general TI case leads to complex forms whereas considering elliptic TI offers very simple forms. For example, Fig. 2 represents the slowness curves of 2D P-waves in isotropic and TTI cases: a circle and a rotated ellipse. Ellipticity here means that the TI coefficients are equal. This hypothesis might be considered as restrictive, but knowing that low-order VTI ABCs only depend on one TI parameter let us think that the simplest TTI ABCs as well.



**Figure 2:** Slowness curves of isotropic (left) and elliptic TTI (right) cases – for 2D P-waves

The construction of the new ABC is then based on a change of coordinate that transforms a circle into a rotated ellipse. The ABC is obtained by applying the coordinate change to the isotropic ABC. We will provide numerical results to assess the performance of the new ABC in TTI elastic media.

The authors acknowledge the support by the Inria-Total strategic action DIP (*dip.inria.fr*).

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