

## INFLUENCE OF BENDING RESISTANCE ON THE DYNAMICS OF A CAPSULE IN SHEAR FLOW

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**Key words:** *capsule, fluid-structure interaction, shell element, finite element method, boundary integral method.*

Capsules that consist of an internal liquid droplet enclosed by a thin hyperelastic wall have numerous applications in bioengineering and pharmaceuticals as biomimetic models of cells such as red blood cells, vectors for drug targeting, etc. When subjected to an external flow, capsules undergo large deformations and, in some cases, buckling because of the strong fluid-structure viscous coupling of the membrane with the internal and suspending fluids. The objective is to study numerically the effect of bending on the dynamics of an initially spherical capsule in shear flow. We analyze how the bending resistance influences the capsule deformation and wrinkle formation.

The numerical method based on [3] couples a shell finite element method for the capsule deformation with a boundary integral method for the low Reynolds-number internal and external flows. It consists in following the position of the nodes of the capsule wall at each time step, which leads to a non conventional implementation of the finite element method. The viscous load exerted by the fluids on the wall is deduced from the wall equilibrium. This method has been shown to be stable in the presence of in-plane compression. The capsule wall is discretized with MITC (Mixed Interpolation Tensorial Components) triangular shell finite elements, based on the shear-membrane-bending model [1]. To model the thin capsule wall behavior, we implement a hyperelastic constitutive law (strain-hardening or strain-softening) for the membrane effects combined with the generalized Hooke's law for the bending effects. The coupling method is validated by simulating the motion of an initially-spherical capsule suspended in a simple shear flow with the same viscosity for the inner and outer fluids. Present results are shown for the generalized Hooke's law. We study the influence of two non-dimensional parameters: the capillary number  $Ca$ , ratio

of the viscous to elastic forces and the bending number  $B$ , ratio of the bending to shear modulus. This parameter can also be considered as the ratio of the membrane thickness to the sphere radius.

For all the values of the bending number ( $B \neq 0$ ), the capsule is elongated in the straining direction at steady state, while the vorticity of the flow induces the rotation of the wall around the steady deformed shape. This motion called *tank-treading* is exactly the same as that observed when the wall of the capsule is modelled with a 2D membrane ( $B = 0$ ) [2, 3]. Even in the case of large bending numbers ( $B = 0.3$ ), the principal axes lengths  $L_1$  and  $L_2$  of the capsule in the shear plane differ by less than 3% between the shell and membrane models (Fig. 1a). For a given  $Ca$ , the capsule has the same average shape in the shear plane as the one predicted when  $B = 0$ . The capsule motion and deformation are thus marginally influenced by the bending stiffness and mainly governed by shear elasticity and membrane effects. When the capsule wall is modelled with a membrane model, wrinkles appear in the central region of the capsule for lower values of  $Ca$  (e.g.  $Ca = 0.1$ , Fig. 1b). They result from the presence of compressive tensions and are in the straining direction. When the bending number is increased, the wrinkle wavelength decreases (Fig. 1c) and for  $B \geq 0.05$ , the wrinkles no longer form (Fig. 1d). Thus, it is possible to prevent buckling with a small bending number.

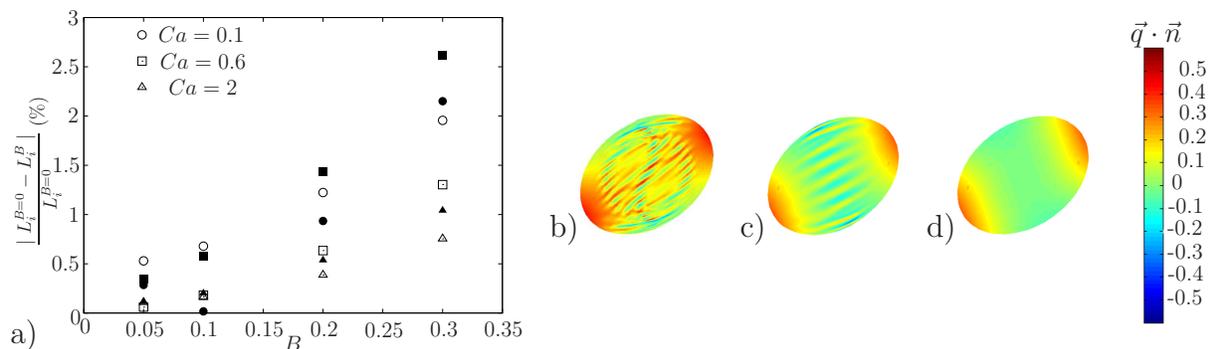


Figure 1: a) Difference in the first and second principal axes lengths of the capsule, respectively  $L_1$  (empty symbol) and  $L_2$  (filled symbol) predicted by the membrane model ( $B = 0$ ) and the shell model. b - d) Steady-state of capsules subjected to a simple shear flow ( $Ca = 0.1$ ) for bending numbers  $B = 0$  b),  $B = 0.01$  c)  $B = 0.05$  d). The color scale represents the normal load.

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