11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

MONOTONICITY IN HIGH-ORDER CURVILINEAR FINITE ELEMENT FIELD REMAP

Robert W. Anderson[1], Veselin A. Dobrev[2], Tzanio V. Kolev[3] and Robert N. Rieben*[4]

[1] CASC, Lawrence Livermore National Laboratory, anderson110@llnl.gov
[2] CASC, Lawrence Livermore National Laboratory, dobrev1@llnl.gov
[3] CASC, Lawrence Livermore National Laboratory, kolev1@llnl.gov
[4] WCI, Lawrence Livermore National Laboratory, rieben1@llnl.gov

Key words: *multi-material hydrodynamics, monotonicity, ALE methods, finite element methods, high-order methods.*

The remap phase in arbitrary Lagrangian-Eulerian (ALE) hydrodynamics involves the transfer of field quantities defined on a post-Lagrangian mesh to some new mesh, usually generated by a mesh optimization algorithm. This problem is often posed in terms of transporting (or advecting) some state variable from the old mesh to the new mesh over a fictitious time interval. It is imperative that this remap process be monotonic, i.e. not generate any new extrema in the field variables. It is well known that the only linear methods which are guaranteed to be monotonic for such problems are 1st order accurate; however, much work has been done in developing non-linear methods which blend both high and low (1st) order solutions to achieve monotonicity and preserve high order accuracy when the field is sufficiently smooth [1]. Here, we consider a set of new methods for enforcing monotonicity targeting high-order Discontinuous Galerkin (DG) methods for advection equations in the context of high-order curvilinear ALE hydrodynamics. Using the general high-order finite element approach described in [2], we consider an ALE formulation based on three phases:

- **Lagrangian phase**, solving the Euler equations on moving curvilinear mesh
- **Mesh optimization phase**, using harmonic or inverse-harmonic smoothing
- **Remap phase**, based on conservative and monotonic DG advection remap

Semi-discrete DG methods for advection equations can be formulated in terms of high-order finite element "mass" and "advection" matrices. These formulations result in high-order accuracy for sufficiently smooth fields, but produce non-monotonic results (spurious oscillations) for discontinuous fields. We present three non-linear approaches for modifying both the mass and advection matrices from the remap phase to enforce monotonicity:

- **Locally Scaled Diffusion (LSD)**: In this approach, we locally lump the mass matrices and upwind the advection matrices using an iterative process and a local "monotonicity measure."
- **Flux Corrected Transport (FCT)**: This is a high-order generalization of the FCT methods of [1] which is modified to account for non-lumped mass matrices.
- **Optimization Based Remap (OBR)**: In this approach, we apply an optimization procedure to enforce monotonicity in the remapped fields, based on the work of [3].

For each approach, we describe the relevant theory and numerical algorithms and demonstrate their effectiveness on standard multi-material ALE hydrodynamics benchmarks, such as the triple-point ALE calculation from Figure 1.

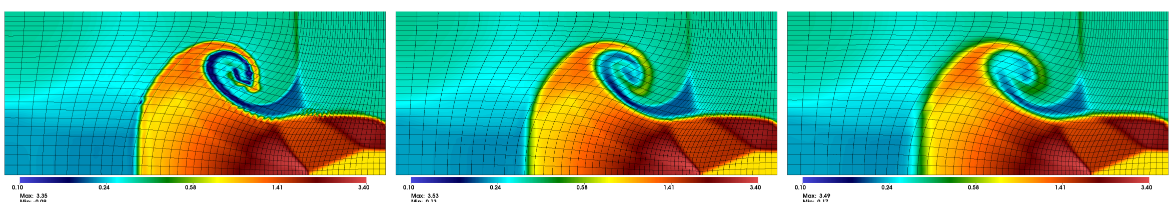

Figure 1: Shock triple-point ALE calculation using Q_4-Q_3 curvilinear finite elements without any monotonicity treatment (*left*), using locally scaled diffusion (*middle*) and using a high-order flux corrected transport algorithm (*right*).

REFERENCES

[1] D. Kuzmin, R. Lohner, and S. Turek. *Flux-Corrected Transport.* Principals, Algorithms and Applications. Springer Verlag, first edition, 2005.

[2] V. A. Dobrev, Tz. V. Kolev, and R. N. Rieben. High order curvilinear finite element methods for Lagrangian hydrodynamics. *SIAM J. Sci. Comp.*, 5(34):B606–B641, 2012.

[3] P. Bochev, D. Ridzal, and K. Peterson. Optimization-based remap and transport: A divide and conquer strategy for feature-preserving discretizations. *J. Comput. Phys.*, 257:1113–1139, 2013.

This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344, LLNL-ABS-648032.