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ADJOINT-BASED ANISOTROPIC *hp*-ADAPTIVE HYBRIDIZED DISCONTINUOUS GALERKIN METHODS FOR TURBULENT FLOW

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Turbulent flow around complex geometries is characterized by a large range of physical scales and strongly anisotropic features. Simulation tools using suitable adaptive strategies promise reduced time-to-solution and higher accuracy per degree of freedom.

The flow solution does not usually represent the final result but is rather an input to a post-processing step in which specific scalar quantities are computed, such as lift or drag coefficients in external aerodynamics. With the aim of computing accurate values for such functional quantities in the most efficient way, target-based error control methods have been developed. One such method is based on the adjoint solution of the governing equations. In this method, an additional linear system is solved which then gives an estimate on the spatial error distribution contributing to the error in the target functional. This estimate can be used as a criterion for local mesh adaptation.

The computational workhorses of industrial-strength compressible flow simulation are usually finite volume methods having at best second order of consistency. However, the requirements on these techniques are continuously increasing, necessitating further work on more accurate and efficient methodologies. Within the context of high-order methods, discontinuous Galerkin (DG) methods have attracted interest. Despite their popular advantages — high-order accuracy on unstructured meshes, a variational setting, and local conservation, just to name a few — they introduce a large number of degrees of freedom with relatively strong coupling, both within elements and across element interfaces. Hybridization may be utilized to alleviate these disadvantages. Here, the globally coupled unknowns have support on the mesh skeleton, i.e. the element interfaces, only. As a result, the global system is reduced in size and improved in terms of sparsity at the same time. The solution in the interior of the elements is then obtained using element-wise reconstruction.

An additional advantage of DG methods is the possibility of using locally varying polynomial degrees. This offers an alternative or rather a supplement to mesh-adaptation. In regions where the solution possesses a high degree of smoothness, increasing the polynomial degree of approximation (p-adaptation) is preferred over local mesh-refinement (h-adaptation). Whenever strong gradients or discontinuities are present, h-adaptation should be used. A combination of both methodologies coupled with an effective strategy to decide between h- and p-adaptation thus results in a highly versatile framework, usually termed hp-adaptation.

Our computational framework already comprises adjoint-based anisotropic mesh- and isotropic hp-adaptation for hybridized and standard DG methods (see Woopen et al. [3], Balan et al. [1] and Woopen et al. [2]). The anisotropic mesh-refinement strategy involves high-order anisotropy-detection and Riemannian metrics. In this work, we extend it to anisotropic hp-adaptation for turbulent flow problems using the k-ω turbulence model. We validate the accuracy of our method and show its robustness with problems stemming from external aerodynamics.

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