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MORTAR-BASED CONTACT FORMULATION WITH ALTERNATIVE CELL PARTITION FOR NUMERICAL INTEGRATION

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The presented segment-to-segment (STS) contact formulation is based on the work of Puso and Laursen [1]. For this and almost all other STS formulations the mortar method is used. This method was originally introduced to tie non-conforming meshes [2]. The main difference compared to simpler node-to-segment (NTS) formulation is the evaluation of contact integrals. Instead of collocating the integration at points, the STS approach uses numerical quadrature to evaluate the contact integrals. The non-penetration condition is met in an integral, weak manner. In order to add the contact conditions the Lagrange multiplier method is used.

The present study proposes a method to improve efficiency of the algorithm via an alternative approach to evaluate the contact integrals which emanate from the underlying STS formulation. These integrals are present in the formulation of the virtual contact work as well as in the computation of the gap. Calculation of the so-called mortar integrals can be done by

$$\mathbf{D}[j,j] = \int_{{\gamma_c^{(1)}}^h} N_j^{(1)} \mathrm{d}\gamma \mathbf{I_3}\,, \qquad \mathbf{M}[k,j] = \int_{{\gamma_c^{(1)}}^h} \Phi_k^{(1)} N_j^{(2)} \mathrm{d}\gamma \mathbf{I_3}\,,$$

with $N_j^{(i)}$ and $\Phi_k^{(i)}$ being shape functions of displacements and contact stresses, respectively. Due to the finite element formulation the contact surface ${\gamma_c^{(1)}}^h$ is faceted and thus the mortar integrals have to be evaluated element by element. For simplicity, the contact surface may be approximated as a piecewise flat surface [1], although in general three-dimensional problems the contacting surface can be curved even for a discretization with bilinear shape functions. Evaluation of mortar integral $\mathbf{M}$ requires an even finer partition because the integration is done on the contact surface of body $\Omega^{(1)}$ but the shape function $N_j^{(2)}$ is defined on the opposite contact surface. Due to the non-smoothness of the integrand segments have to be found on which the integrand is smooth.

Puso and Laursen [1] propose to further subdivide the segments into triangular integration cells, which are constructed by connecting the centre of gravity of the segment with its corners. This way one n-polygonal segment is subdivided into n integration cells. Numerical quadrature with seven integration points is applied to each integration cell.

In order to reduce the effort needed for integration, alternative integration cells are investigated here. Segments are constructed as before, but quadrilateral integration cells are used whenever possible. To avoid integration cells with very small angles a Delaunay triangulation is applied first. Each quadrilateral integration cell is evaluated using nine integration points. Thus, polynomials up to the fifth order can be integrated exactly as for the triangular cells. For general geometries this accuracy is needed due to the mapping of the integration points to the slave and master side, which can result in rational polynomials. In those cases integration will never be exact; however, for the numerical tests performed so far, the accuracy seems to be sufficient.

Introduction of quadrilateral integration cells reduces the number of integration points, which yields a more efficient formulation. In the case of a segment with eight nodes only 27 instead of 56 integration points are needed with the proposed method. Numerical examples show a reduction of the overall computation time by almost 50 % maintaining nearly the same accuracy. Results of this method will be compared to other approaches.

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