11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

Automated assessment of anisotropic elasticity of hard tissue samples using resonant ultrasound spectroscopy with Bayesian analysis and Monte Carlo methods

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Key Words: *Inverse problem, Bone, Vibrations, Gibbs sampling, Simulated annealing.*

Biological hard tissues such as cortical bone and dental tissues exhibit remarkable mechanical properties at organ-scale, due to a highly organized tissue structure at smaller length scales. Because many pathologies affect tissue organization, a better understanding of the relationships between structure and mechanical properties is desirable, and the assessment of anisotropic elasticity at the millimeter scale is an important part of the investigation.

Proper characterization of the anisotropic elasticity at the millimeter scale requires measurements of all the terms of the elastic tensor on a unique small volume of material. Resonant Ultrasound Spectroscopy (RUS) uses the free vibration spectrum of a sample to estimate elasticity, based on a comparison of measured and model-predicted resonant frequencies [1]. This method was developed for crystals and metals but has been recently applied to bone [2]. The difficulty in that application is the high level of mechanical damping, which causes resonant peaks to overlap. Some predicted frequencies cannot be observed and one faces the problem of finding which predicted frequencies compare to the measurements, or in other words, of correctly pairing the predicted and measured frequencies in the definition of a cost function. In this work we propose a Bayesian formulation and the use of Monte Carlo Markov chain (MCMC) methods to overcome this problem.

Pairing of predicted and measured frequencies is represented by vector A. Probability distributions over the elastic parameters θ and pairing A are defined. The posterior joint probability of parameters and pairing given the data y, $p(\theta, A \mid y)$, which is the solution of the problem, is explored using Gibbs sampling, which successively samples the two conditional distributions $p(A \mid y, \theta)$ and $p(\theta \mid y, A)$. Samples from the first conditional distribution are obtained from a MCMC with simulated annealing while samples from the second conditional distribution are obtained from a multivariate normal approximation.

This approach allows proper inclusion of prior information on material elasticity, and inclusion of experimental uncertainties on frequency measurement. In contrast with other approaches in RUS, our approach doesn't require to carefully pair the frequencies using trial-and errors procedure [1] or sensitivity analysis [3]. Results on elastic properties are summarized by the moments of the marginal distribution $p(\theta \mid y)$, which is an average over different probable pairings. This makes the method more robust and automated. Applicability is demonstrated on experimental data sets for isotropic and anisotropic synthetic materials, for which the solution is known, and on data sets for cortical bone samples.

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