

## ISOGEOMETRIC ONE-PARAMETER FORMULATIONS FOR SHEAR DEFORMABLE STRUCTURES

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We present formulations for shear deformable structures such as Timoshenko beams and Reissner-Mindlin plates with only one unknown variable, similar to the shear-rigid formulations, i.e., Bernoulli beams and Kirchhoff plates. While in formulations based on the shear-rigid theories rotations of the cross section are typically obtained as derivative or gradient of the deflection, this is not the case for formulations based on the shear-deformable theories. Due to shear deformability, deflections and rotations are typically considered as decoupled, yielding equation systems with two unknowns. Such two-parameter formulations are generally considered as the irreducible form of the corresponding partial differential equations.

In this presentation we show that the partial differential equations for shear deformable structures can be reduced to one unknown through a consequent exploitation of the underlying governing equations without additional assumptions. It is argued that the traditional formulations with two unknowns are mainly motivated by considerations regarding discretization with linear finite elements and are not a necessary result of the underlying theories. We derive the strong form equations with only one unknown and from this we develop weak forms using Galerkin's method, which are used as basis for numerical implementations. Choosing the test functions appropriately, we obtain the classic expressions of virtual work with bending and shear terms, with the difference that all appearing variables are expressed in terms of one unknown. These formulations exclude the typical problem of shear locking *ab initio* and are considerably more efficient compared to standard formulations due to the reduced number of degrees of freedom. Since higher order derivatives appear in the variational formulations, high-order and high-continuity elements are necessary, such as isogeometric NURBS-elements. We show the derivation as well as the numerical implementation for both beams and plates. Numerical tests confirm the validity and good performance of the presented approach.