

# HIERARCHICAL MESH SMOOTHING AND UNTANGLING FOR TWO AND THREE DIMENSIONAL MESHES

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It is well known that the accuracy of a finite element method calculation depends on the quality of the domain discretization. For this reason, mesh smoothing and untangling is a key process to improve the accuracy of a simulation. Typically, when a smoothing algorithm is applied, the boundary nodes are considered fixed, and the method obtains the *best* position of the interior nodes to improve the mesh quality [1]. However, the mesh boundary may constrain the quality of the whole mesh, specially when thin regions are present, and low-quality elements can be obtained.

To overcome this issue, we propose a hierarchical smoothing and untangling process based on minimizing an objective function defined as the distortion of the three dimensional elements, and using the nodal coordinates as arguments [2]. The novelty of this work is that we use a single objective function that also depends on the position of the boundary nodes. This allows using a hierarchical approach in the optimization process. First, we improve the quality of the three-dimensional elements moving the nodes located on the edges along them. Second, we improve the quality of the three-dimensional elements moving the nodes on the surfaces taking into account that these nodes have to be located on the surface. Finally, we minimize the mesh distortion moving the inner nodes. During the first two steps of the proposed method, we have to ensure that the boundary nodes are located on the corresponding edges and surfaces. To this end, we consider the parameterization of the edges and surfaces of the CAD model. Specifically, the unknowns that correspond to the edge and surface nodes are determined by their parametric coordinates on the CAD entities. In this manner, we can avoid expensive and non-robust projections of the nodes on the CAD entities. Furthermore, we propose an object oriented smooth-

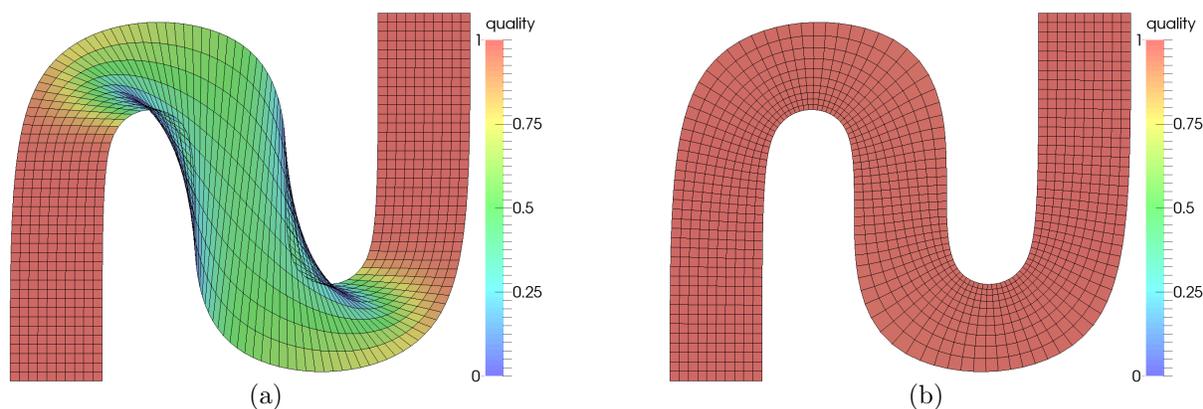


Figure 1: Simple quadrilateral mesh: (a) before applying the hierarchical smoothing; and (b) after applying the hierarchical smoothing.

ing environment to untangle and improve the quality of meshes composed by: triangles, quadrilaterals, tetrahedra and hexaedra.

The main advantage of the proposed approach is that the location of the edge and surface nodes is not fixed. This results in an optimization process that is less constrained than the standard approach (fixed boundary nodes) and therefore, a better configuration can be achieved. To illustrate this advantage and the applicability of the proposed approach, we present several smoothed and untangled meshes.

## REFERENCES

- [1] P.M. Knupp, Achieving finite element mesh quality via optimization of the Jacobian matrix norm and associated quantities. Part I - A framework for surface mesh optimization. *International Journal of Numerical Methods in Engineering*, Vol. **48**, 401–420, 2000.
- [2] A. Gargallo-Peiró, X. Roca, J. Peraire and J. Sarrate. A surface mesh smoothing and untangling method independent of the CAD parameterization. *Computational Mechanics*, 2013.