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INVESTIGATION OF HIGH-ORDER TEMPORAL SCHEMES FOR THE DISCONTINUOUS GALERKIN SOLUTION OF THE NAVIER-STOKES EQUATIONS

F. Bassi[1], C. De Bartolo[2], N. Franchina[1], A. Ghidoni[3,*] and A. Nigro[2]

[1] Department of Industrial Engineering, University of Bergamo, viale Marconi, 5 - 24044 Dalmine (BG), Italy. Email: francesco.bassi@unibg.it, nicoletta.franchina@unibg.it

[2] Department of Mechanical, Energetic and Management, University of Calabria, via P. Bucci, 44/C - 87036 Rende (CS), Italy. Email: carmine.debartolo@unical.it, alessandra.nigro@unical.it

[3] Department of Mechanical and Industrial Engineering, University of Brescia, Via Branze, 38 - 25123 Brescia, Italy. Email: antonio.ghidoni@ing.unibs.it

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In recent years Discontinuous Galerkin (DG) methods have received increasing attention in Computational Fluid Dynamics (CFD) [1] due to many attractive features, such as the geometrical flexibility, the use of elements with different solution polynomial approximation in the same grid and the compactness of the scheme. However, these methods have also an high computational cost and memory requirement. The DG space discretized equations can be advanced in time using different time integration schemes. Explicit Runge-Kutta methods are very popular for the solution of unsteady problems characterized by strong discontinuities and/or fast space-time oscillations, and can match in time the high accuracy of the DG discretization. These schemes can be very inefficient due to the time-step restriction for turbulent simulations. Implicit time integration schemes can be adopted to overcome this limitation, such as the multistep Backward Differentiation Formulae (BDF). However, BDF are only A-stable up to the second-order and $A(\alpha)$-stable up to order 9 and their low accuracy is not well suited to match the spatial accuracy of DG methods. In this work different high-order temporal schemes will be analyzed: the Explicit Singly Diagonally Implicit Runge Kutta (ESDIRK) [2], the Modified Extended BDF (MEBDF) [3], the Two Implicit Advanced Step-point (TIAS) [4] and a Rosembrock method [5]. The proposed schemes have been evaluated for two unsteady test-cases: (i) the convection of an inviscid isentropic vortex and (ii) the laminar flow around a cylinder. The accuracy and the design-order convergence are assessed in terms of the L^2 norm of the pressure error, using the analytical and a reference solution for the vortex and the cylinder test case, respectively. High-order and standard second order BDF (BDF2) schemes are compared in terms of CPU time needed to reach a given accuracy to show when high-order temporal schemes can be advantageous. Figure 1 shows the potential of the sixth order TIAS (TIAS6) scheme in comparison with the BDF2 for the cylinder test case. The density and velocity contours are depicted after 30 vortex shedding periods, highlighting the greater accuracy provided by the TIAS6 scheme.

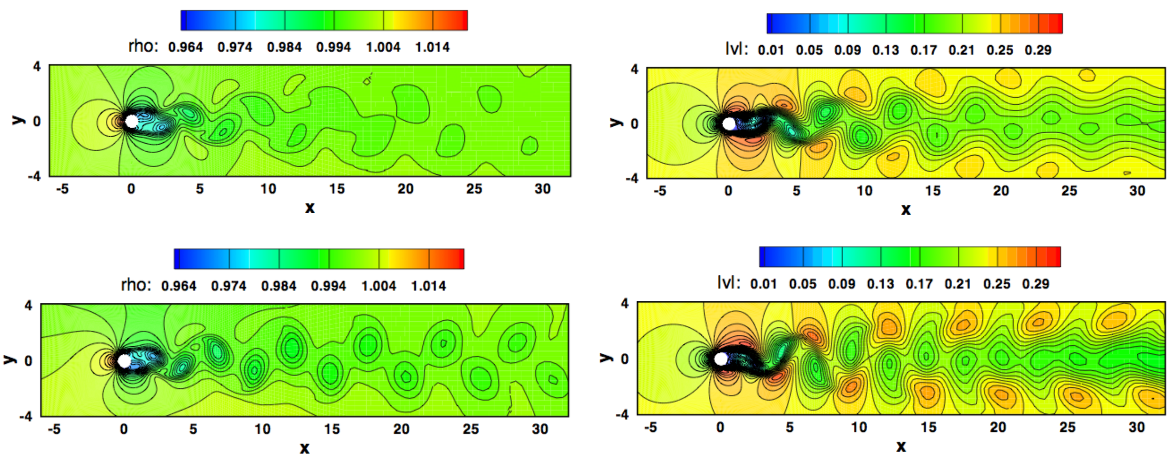

Figure 1: Density (left column) and velocity magnitude (right column) fields after 30 vortex shedding periods. Top row: P5-BDF2. Bottom row: P5-TIAS6.

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