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AN ADJOINT APPROACH FOR INVERSE ANALYSIS IN PHOTOACOUSTIC IMAGING USING THE HYBRIDIZABLE DISCONTINUOUS GALERKIN METHOD

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Photoacoustic tomography is a medical imaging technique which combines diffuse optical tomography and ultrasonography. An object of interest is illuminated by a short time laser pulse and the light propagates within the object according to its optical properties. At areas of high absorption, the optical energy is transformed into heat followed by thermal expansion. This expansion initiates pressure waves which are eventually measured by acoustical detectors. The recorded acoustical signal allows for conclusions on optical and mechanical properties of the underlying material. The explanatory power of the recorded signals depends among others on the accuracy of the used reconstruction algorithm [1].

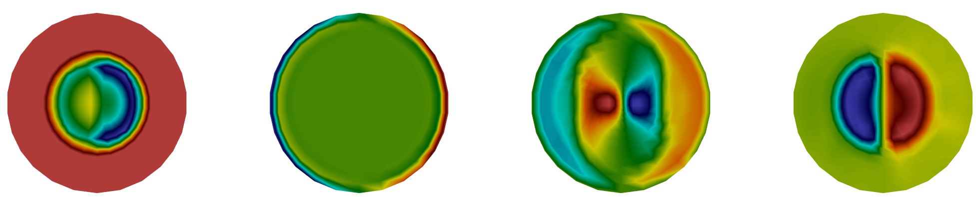
This presentation concentrates on the efficient simulation of wave propagation in heterogeneous media with the hybridizable discontinuous Galerin (HDG) method, and the formulation of the inverse problem by use of the adjoint method for the discretized photoacoustic problem.

The HDG method is a class of discontinuous Galerkin (DG) methods which significantly reduces the number of global unknowns to be solved for implicit time integration schemes, as compared to DG, by introducing the trace field as a new variable [2]. The trace is used to define the numerical flux and, in contrast to the discontinuous field variables, is single-valued on element faces. The HDG formulation is especially well suited for high-order polynomial bases and hence competitive to e.g. partition of unity or ultra weak variational methods and superior to the classical DG or the standard continuous Galerkin method for wave propagation problems. Also, the method naturally treats heterogeneous materials as opposed to methods relying on the fundamental solutions of the wave equation.

The inverse algorithm estimates the optical and acoustic material properties based on measured pressure curves. To calculate the gradient of the objective function, the adjoint method described in [3] is applied to the photoacoustic problem. The derivation of the dual problem is based on the spatially discretized Lagrangian to properly include the stabilization terms inherent in the HDG formulation of the acoustic wave equation. Also, special attention is required for the choice of adequate time integration schemes. Immoderate dissipative errors in combination with high frequency excitation give rise to deviations in the gradient calculation.

A reconstruction algorithm is proposed which is based on a simple BFGS method and consists of the calculation of the primal problem, the calculation of the dual problem driven by the difference between measured and simulated pressure time curves, the evaluation of the gradient with respect to the discrete parameters, and a line search to ensure satisfaction of the Wolfe conditions. When describing the material distribution via parametrized regions, the discrete parameter space contains typically only a few tens of variables. For more accurate results, the reconstruction algorithm can also directly be applied to find the full material distribution in the discretized domain. In this case, the parameter space gets large and the use of a limited memory BFGS becomes relevant.

Figure 1 shows pressure fields for the primal and the dual problem at various time steps for a numerical example of a photoacoustic setting. The initial pressure field of the primal problem as in 1(a) results from the simulation of the light propagation. After time integration of the primal problem and monitoring of the pressure, the dual problem is integrated backwards in time with a source term at the boundary 1(b)–1(d). From the pressure field at initial time 1(d), the dual light energy distribution is calculated and the gradient is evaluated accordingly. Examples with experimentally obtained data will verify the applicability of the developed method.



(a) Primal solution at $t = 0$s (b) Dual solution at $t = 5 \cdot 10^{-6}$s (c) Dual solution at $t = 1.4 \cdot 10^{-6}$s (d) Dual solution at $t = 0$s

Figure 1: Pressure field for primal and dual problem at various time steps

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