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STABILITY ASPECTS OF PRESSURIZED MEMBRANES

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We have studied the response to pressurization in thin membranes, primarily closed ones. The intended usage is in biological and medical contexts, but polymeric membranes also have many technological applications. The considered areas suggest that the membranes are subjected to pressurization from gases or fluids, with medium density and compressibility as distinguishing aspects. The precise method for introducing the pressure, via specified pressure or amount of the medium, has been shown to be important in stability investigations. So will for many practically interesting cases the pressure-expansion relation show a limit-point instability, with a snap-through behavior, whereas the relation between gas amount and expansion will be monotonous, [1].

Membranes have been analytically modelled, but most of the results presented have come from finite element based simulations. These have used a triangular element, with a local plane-stress assumption, i.e., neglecting the bending part of the stiffness. The formulation can use different incompressible hyper-elastic material models in a large strain context. The triangular element will thereby have constant stress and strain conditions in its internal. The assembled structure is given in a Total Lagrangean framework, using only translation degrees of freedom. Unless all elements connected to a node are co-planar, the geometrical non-linearity can create a stable response.

Studies have been directed towards the hyper-elastic material model. We have shown that the commonly used two-parameter Mooney-Rivlin model can show a critical response when a element model is subjected to bi-axial strains. This critical response, numerically similar to a buckling bifurcation, can lead to non-unique deformations above a certain stress level, when edge loads are prescribed based on reference geometry. They disappear for loadings related to current geometry. From a pragmatic viewpoint, the movability of the loading can be seen as having a stabilizing effect. We have shown that this form of critical response can not exist for material models where only the first invariant of strain is used in the strain energy density function, but that other hyper-elastic models could share the problem. Our work has given a general method for investigation of these material models, [2].

Previously, the response to gas pressurization has shown that balloon-like structures can give limit point and bifurcation instabilities. We recently have focussed on the pressurization by a fluid, e.g., water. These simulations show that other forms of instabilities –or strongly non-linear effects– will result from the hydrostatic pressure distribution, in particular when a membrane is only partially filled with fluid. Also for these loadings, the constitutive constants in the used hyper-elastic model play important roles in not only the quantitative but also the qualitative response. Simulations further show that a structural model can pass between different externally unloaded configurations through a loading sequence. Unless careful discretization is introduced, the response and instabilities of the pressurized membrane can be strongly affected by the element mesh.

Pressurization with fluids also set a main focus on the membrane wrinkling problem. Compared to a gas-based pressurization, where almost any initial structural configuration will tend to a uniformly tensioned sphere, the partially fluid-filled membranes will often consist of compressed parts, where wrinkling can occur. Wrinkling, which can not be fully resolved even with fine element meshes, and for which the mechanical criteria are not fully understood, is an important aspect of membrane simulations, and some illustrative examples will be shown. We will show how different wrinkling criteria from literature will lead to different conclusions for simple flat or shallow membranes subjected to hydrostatic pressure. We will also describe some tentative efforts for handling wrinkling in a computational model.

As pressurized membranes are often completely defined by a low number of design parameters, their optimization can be seen in an uncommon setting, when the constraints on response, i.e., on the displacements, stresses or instability levels, can be traced by generalized path-following in a parameterized space. This view reduces the problems encountered with most optimization algorithms when dealing with severely non-linear structures, e.g., their non-unique responses to loading, [3].

REFERENCES

[1] A. Eriksson and A. Nordmark. Instability of hyper-elastic balloon-shaped space membranes under pressure loads. *Computer Methods in Applied Mechanics and Engineering*, Vol. **237–240**, 118–129, 2012.

[2] A. Eriksson and A. Nordmark. Non-unique response of Mooney-Rivlin model in biaxial membrane stress. Manuscript 2013.

[3] A. Eriksson. Constraint paths in non-linear structural optimization. Manuscript 2013.