11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

UNIVERSAL MESHES FOR PROBLEMS WITH MOVING BOUNDARIES

Evan S. Gawlik[*1] **and Adrian J. Lew**[2]

[1] Computational and Mathematical Engineering, Stanford University, Stanford CA 94305, egawlik@stanford.edu

[2] Mechanical Engineering, Stanford University, Stanford CA 94305, lewa@stanford.edu

Key words: *Moving boundary, error analysis, ALE, universal mesh*

We develop a framework for the design of finite element methods for moving boundary problems with prescribed boundary evolution that have arbitrarily high order of accuracy, both in space and in time. At the core of our approach is the use of a universal mesh: a stationary background mesh containing the domain of interest for all times that adapts to the geometry of the immersed domain by adjusting a small number of mesh elements in the neighborhood of the moving boundary. The resulting method maintains an exact representation of the (prescribed) moving boundary at the discrete level, yet is immune to large distortions of the mesh under large deformations of the domain. The framework is general, allowing one to achieve any desired order of accuracy in space and time by selecting a suitable finite-element space on the universal mesh and a suitable time integrator for ordinary differential equations.

In the process of deriving our method, we present a unified framework that puts our method and conventional deforming-mesh methods on a common footing suitable for analysis. In particular, we study a model parabolic problem posed on a moving domain with prescribed evolution, discretized in space with a finite element space that is associated with a moving mesh that conforms to the domain at all times, and discretized in time with a single-step numerical integrator for ordinary differential equations. The moving mesh is assumed to evolve smoothly in time, except perhaps at a finite number of temporal nodes where the solution is transferred between finite element spaces via a projection.

A key result of our analysis is an abstract estimate for the L^2-norm of the error between the exact and numerical solutions at a fixed positive time T, expressed in terms of the difference between the exact solution and its elliptic projection onto the finite element space at times $t \leq T$ [1]. Upon specializing the abstract estimate to particular choices of the mesh motion strategy, finite element space, time integrator, and projector, we derive error estimates in terms of the mesh spacing and time step for various schemes. The schemes we consider include classical ALE schemes as well as schemes based upon universal meshes [2, 3, 4]. We verify the aforementioned error estimates with several numerical examples in one and two dimensions.

REFERENCES

[1] E. S. Gawlik and A. J. Lew. Unified analysis of finite element methods for problems with moving boundaries. Preprint, 2014.

[2] E. S. Gawlik and A. J. Lew. High-order finite element methods for moving-boundary problems with prescribed boundary evolution. Submitted, 2013.

[3] R. Rangarajan and A. J. Lew. Universal Meshes: A new paradigm for computing with nonconforming triangulations. Submitted, 2013.

[4] R. Rangarajan and A. J. Lew. Analysis of a method to parameterize planar curves immersed in triangulations. *SIAM Journal on Numerical Analysis*, Vol. **51**, No. 3, 1392-1420, 2013.