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OPTIMISATION OF THE POSITION OF THE MATERIAL POINTS IN THE MAXIMUM ENTROPY INTERPOLATION

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The maximum entropy interpolation can be used in meshless methods to model problems involving finite strains and large deformations[1]. The particularity of this interpolation is that it is based on a probabilist approach to build a shape function for each node of a given node set. Those shape functions are integrated all over the convex domain $convX$ [2] generated by the node set X using material points. Theoretically, these material points can be put everywhere in the domain $convX$. In this paper we look for optimal positions for these material points.

The maximum entropy interpolation is used to build shape functions defined over a convex domain $convX$ generated by a given node set $X = \{\mathbf{x}_a = (x_a, y_a), a = 1, ..., N\}$. $convX$ is the smallest convex space containing X. As X is finite, $convX$ is convex and compact. Let $u : convX \to \mathbb{R}$ whom values $\{u_a = u(x_a), a = 1, ..., N\}$ are known on X. We want to build approximations $p_a : convX \to \mathbb{R}$ of u.

Arroyo and Ortiz [3] established the expression of the shape functions for a Maximum Entropy interpolation:

$$p_{\beta a} = \frac{1}{Z(\mathbf{x}, \boldsymbol{\lambda}^*(\mathbf{x}))} \exp[-\beta|\mathbf{x} - \mathbf{x}_a|^2 + \boldsymbol{\lambda}^*.(\mathbf{x} - \mathbf{x}_a)] \tag{1}$$

where:

$$Z(\mathbf{x}, \boldsymbol{\lambda}) = \sum_{a=1,}^{n} \exp[-\beta|\mathbf{x} - \mathbf{x}_a|^2 + \boldsymbol{\lambda}.(\mathbf{x} - \mathbf{x}_a)] \tag{2}$$

$$\boldsymbol{\lambda}^*(\mathbf{x}) = \arg \min_{\boldsymbol{\lambda} \in \mathbb{R}^d} \log Z(\mathbf{x}, \boldsymbol{\lambda}) \tag{3}$$

Array $\boldsymbol{\lambda}$ corresponds to Lagrange multipliers imposing conditions of $(LME)_\beta$ problem and for all $\beta \in [0, +\infty[$ and for all $\mathbf{x} \in \text{conv}X$, the minimizer $\boldsymbol{\lambda}^*(\mathbf{x})$ is unique. The objective is to find a set M defined by

$$M = \{\mathbf{x} \in convX \setminus \delta convX | \boldsymbol{\lambda}(\mathbf{x}) = 0\} \tag{4}$$

If we use elements of M as material points, then we do not need the optimisation to find out the value of $\boldsymbol{\lambda}^*$ and since this optimisation represents a non-negligeable computation time, we can simply skip this step since the shape functions are only computed on the material point and the $\boldsymbol{\lambda}$ will always be equal to zero during the computation. The idea is to put integration points at those positions in order to simplify the shape functions evaluation. The determination of the set M is based on the expression of $\boldsymbol{\lambda}^*(\mathbf{x})$. A point $x \in M$ if

$$\boldsymbol{\lambda}^*(\mathbf{x}) = \arg \min_{\boldsymbol{\lambda} \in \mathbb{R}^d} \log Z(\mathbf{x}, \mathbf{0}) = \mathbf{0} \tag{5}$$

Since the minimizer $\boldsymbol{\lambda}^*(\mathbf{x})$ is unique, points in M are also solutions of system (6).

$$\frac{\partial \log Z}{\partial \boldsymbol{\lambda}}(\mathbf{x}, 0) = 0 \tag{6a}$$

$$\frac{\partial^2 \log Z}{\partial \boldsymbol{\lambda}^2}(\mathbf{x}, 0) \geq 0 \tag{6b}$$

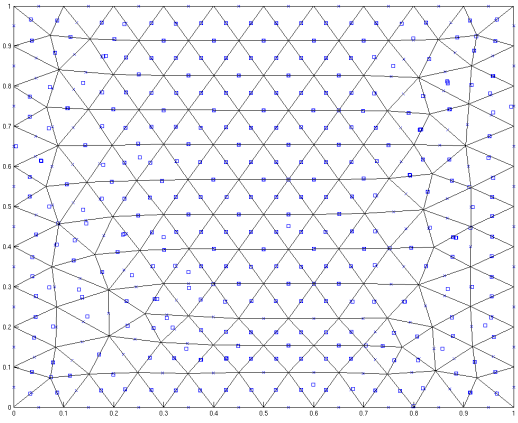

Figure - Optimal position of the material points: initial position x and optimised position □.

An important point to notice is that we want to place material point where $\boldsymbol{\lambda} = 0$ but not put one material point any where $\boldsymbol{\lambda} = 0$. Indeed, we do not need so many material points to integrate the shape function all over the domain correctly so we will not need to find all the solutions of equation (5). We assume that such an optimisation may provide a gain in computational time and solution quality. This method will be applied to thermo-mechanical processes such as friction welding modelisation.

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