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Mixed Meshless Local Petrov Galerkin (MLPG) Collocation Method for Modeling of Heterogeneous Materials

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In recent time, a class of numerical approaches known as meshless methods has attracted considerable attention due to its potential to overcome time-consuming mesh generation and element distortion problems associated with the finite element method. Despite the recent popularity of meshless methods in the scientific community, high numerical costs associated with the calculation of meshless approximation functions still represent serious obstacles. The mixed Meshless Local Petrov-Galerkin (MLPG) Method paradigm [1] represents an efficient remedy for these deficiencies, and has been successfully applied for solving certain demanding engineering problems [2].

In the present contribution, the MLPG formulation based on the mixed approach, which has been efficiently used for the analysis of homogeneous structures [1] is extended for the modeling of deformation responses of heterogeneous materials. Heterogeneous structures consist of various homogeneous subdomains which are discretized by grid points, where equilibrium equations may be imposed. Independent variables are approximated using meshless interpolation functions in such a way that each subdomain is treated as a separate problem according to [3]. The solution for the entire domain is then obtained by gluing the solutions for displacements and tractions along the interfaces of the subdomains by enforcing the corresponding continuity conditions.

Here the collocation meshless method will be applied, which may be considered as a special case of the MLPG approach [4], where the Dirac delta function is employed as the test function. The linear elastic boundary value problem for each subdomain is discretized by using the independent interpolations of both displacements and stress components. The interpolating moving least squares (IMLS) approximation scheme [2] and the radial point interpolation method [5] will be applied. They possess the interpolation property at the nodes, which enables a simple enforcement of the essential boundary conditions (BCs). In order to derive the final closed system of discretized governing equations with the displacements as unknown variables, the nodal stress values are expressed in terms of the displacement components using the kinematic and constitutive relations analogous to the formulation in [2]. The proposed mixed MLPG formulation is compared with the standard fully displacement (primal) formulation in the example considering homogeneous thick cantilever beam with the dimension of $L \times H = 24 \times 4$, as shown in Figure 1. The geometry, discretization, and boundary conditions described by displacements $\bar{u}_x$, $\bar{u}_y$ and tractions $\bar{t}_x$, $\bar{t}_y$ are presented.

The material data are Young's modulus $E = 1$ and Poisson's ratio $\nu = 0.25$. The convergence study of both primal (PA) and mixed (MA) approaches employing the relative error r_u in the L_2 norm of nodal displacements is shown in Figure 2. The nodal distance of the uniform grid is denoted by h. The meshless interpolation schemes using the second- and third-order basis (IMLS2 and IMLS3) are applied and compared.

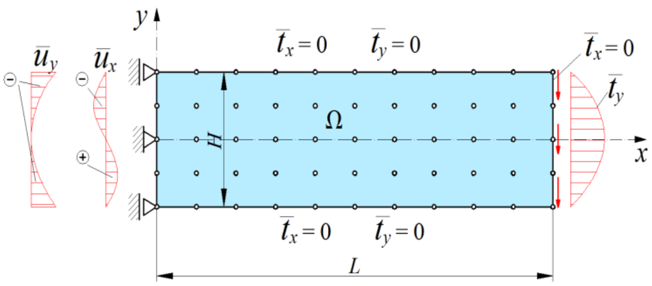

Figure 1: Geometry and BCs of thick cantilever beam

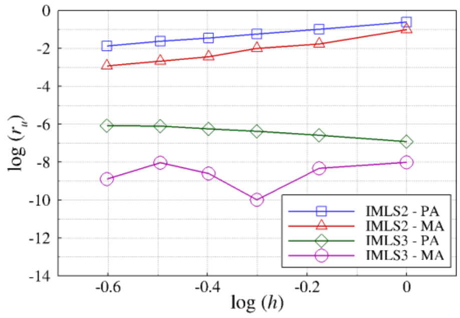

Figure 2: Results of convergence tests

As evident, the mixed approach is superior to the primal formulation. Therefore, a more accurate and numerically efficient modeling of heterogeneous material is expected when the mixed meshless collocation formulation is used. The discretization of the heterogeneous cantilever beam, with geometry and boundary condition described above, consisting of two homogeneous subdomains Ω^+ and Ω^- with different material data is shown in Figure 3. This example will be used as one of the benchmarks for testing the computational strategy which will be presented in the proposed contribution.

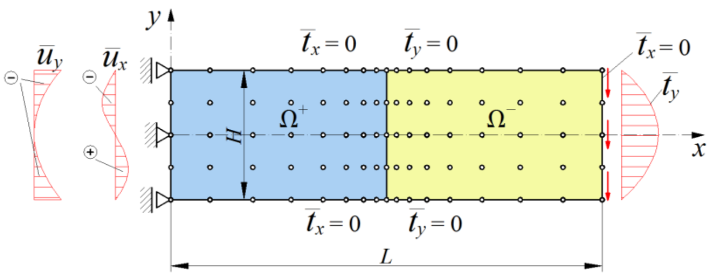

Figure 3: Heterogeneous cantilever beam

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