

# ON THE USAGE OF FINITE DIFFERENCES FOR THE DEVELOPMENT OF DISCRETE LINEARISED AND ADJOINT CFD SOLVERS

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In the framework of the flow solver TRACE developed at DLR, a time-linearised [3] and an adjoint [1] solver have been implemented using a discrete approach [2]. Starting from a finite-volume discretisation of the Reynolds-averaged Navier–Stokes equations, the corresponding discrete linearised or adjointed equations are constructed and solved. This requires the computation of the flux Jacobian, which is done using central finite differences.

Recently, the TRACE code has been differentiated using Algorithmic Differentiation techniques [4]. In this study, we use algorithmically differentiated routines to compute the flux Jacobian for the linear or adjoint solver and compare the results to those obtained using the finite difference approximation. We also investigate the influence of the parameters used in computing the finite differences, in particular the step size. Moreover, we compare different choices for the slope limiter in the numerical scheme, since this is a somewhat critical issue when differentiating a flow solver.

As a first example, we use a rather simple test case, where the linear solver is used to simulate the transport of an acoustic wave through a duct (see Fig. 1). Depending on the frequency of the wave and the solver settings, different amounts of numerical dissipation can be observed (Fig. 2).

We see in Fig. 2(a) that, if no limiter is used, the results using finite differences agree well with those obtained using the exact flux Jacobian, as long as the step size is not too large. In contrast, Fig. 2(b) shows that for one choice of the limiter function we can not reach the exact result even with a reasonable value of the differentiation parameters. Instead, the observed dissipation corresponds to that obtained when using a first order spatial discretisation scheme.

In the final paper, we also discuss to what extent these findings are transferable to applications in the context of turbomachinery design, in particular sensitivity computations using the adjoint solver.

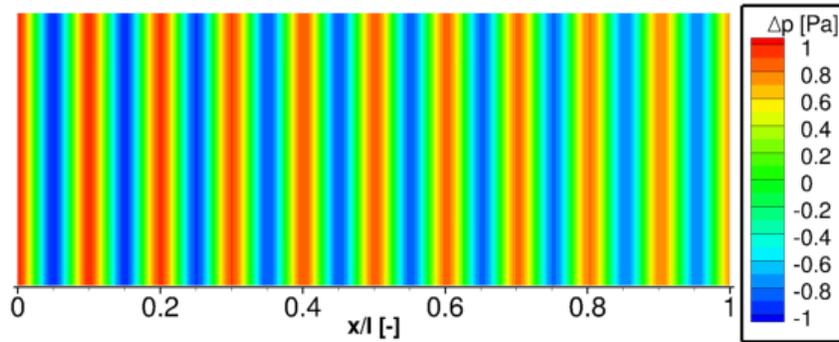
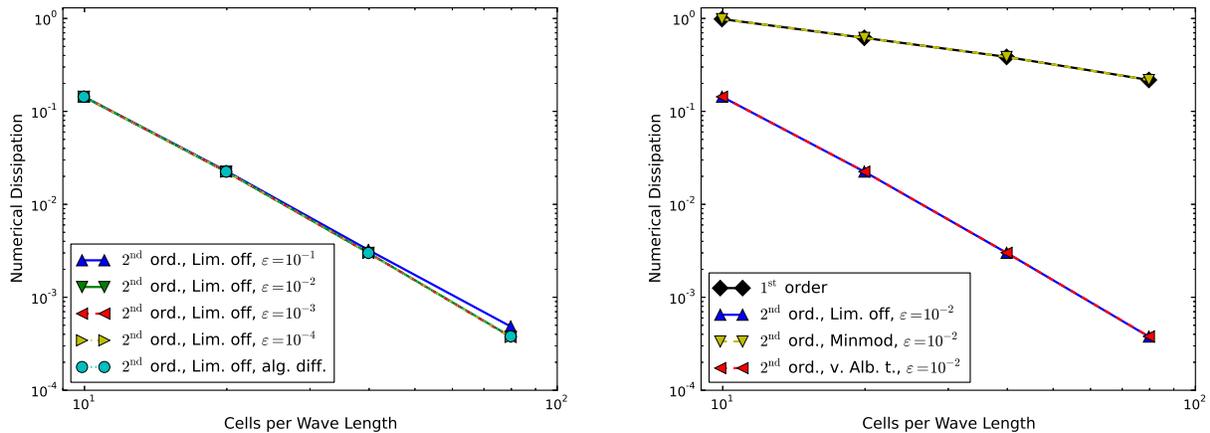


Figure 1: Solution for the propagation of a wave with a frequency of 5000 Hz produced by the linear solver with the following settings: second order spatial discretisation, van Albada type limiter, step size  $\varepsilon = 10^{-2}$  for finite differences



(a) Variation of differentiation method/step size

(b) Variation of limiter/spacial accuracy

**Figure 2:** Numerical dissipation as function of frequency computed using different solver setups

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