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THERMAL STRESS OSCILLATION BEHAVIOR IN A FUNCTIONALLY GRADED MATERIAL THIN FILM

Fumihiro Ashida[1]* and Takuya Morimoto[2]

[1] Shimane University, Matsue, Shimane 690-8504, Japan, ashida@ecs.shimane-u.ac.jp
[2] Shimane University, Matsue, Shimane 690-8504, Japan, morimoto@riko.shimane-u.ac.jp

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INTRODUCTION

Numerous papers on dynamic or generalized thermoelastic problems in functionally graded material (FGM) solids as well as in nonhomogeneous solids have been published. Most of those, however, dealt with harmonic wave propagation behaviors and short-time responses to nonharmonic loadings. Long-time solutions to dynamic or generalized thermoelastic problems of FGM or nonhomogeneous solids subjected to nonharmonic loadings were obtained in a limited number of papers, but they only showed that monotonic or complicated stress oscillations were induced in the solids.
In the present paper, a one-dimensional dynamic thermoelastic problem of an FGM infinite thin film subjected to a thermal shock loading is analyzed, when variations in material properties of the thin film are expressed as exponential functions of the space variable. Exact analytical solutions are obtained for two cases of variations in material properties. The time history of one stress is periodically monotonic and similar to it derived for a homogeneous thin film, whereas that of the other stress is unsteady and the amplitude changes considerably with time. The factor, which governs whether a stress oscillation is monotonic or unsteady, is also investigated.

ANALYSIS

Let us consider a one-dimensional dynamic thermoelastic problem in an FGM infinite thin film of the thickness l, as shown in Fig. 1. It is assumed that the thin film is considered to be initially at the reference temperature, the top surface is suddenly exposed to the uniform temperature rise T_C, and the bottom surface is kept at the reference temperature:

$$T = 0 \text{ at } t = 0, \quad T = T_c \text{ on } z = 0, \quad T = 0 \text{ on } z = l \quad (1)$$

where $T(z,t)$ is the temperature change from the reference temperature and t is time. The temperature field in the FGM thin film is governed by

$$\frac{\partial}{\partial z} k(z) \frac{\partial T}{\partial z} = C(z)\rho(z)\frac{\partial T}{\partial t} \quad (2)$$

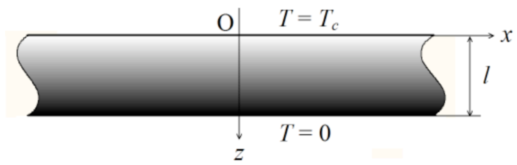

Fig. 1 An FGM thin film

where $k(z)$ is the thermal conductivity, $C(z)$ is the specific heat, and $\rho(z)$ is the density.
It is assumed that the thin film is initially at rest and both surfaces are stress-free as follows:

$$u_z = \frac{\partial u_z}{\partial t} = 0 \text{ at } t = 0, \quad \sigma_{zz} = 0 \text{ on } z = 0, \quad \sigma_{zz} = 0 \text{ on } z = l \quad (3)$$

where $u_z(z,t)$ is the displacement and $\sigma_{zz}(z,t)$ is the stress. The constitutive equation is given by

$$\sigma_{zz} = \{\lambda(z) + 2\mu(z)\}u_{z,z} - \{3\lambda(z) + 2\mu(z)\}\alpha(z)T \quad (4)$$

where $\lambda(z)$ and $\mu(z)$ are Lamé's constants and $\alpha(z)$ is the coefficient of linear thermal expansion. The equation of motion with inertia is expressed as:

$$\frac{\partial \sigma_{zz}}{\partial z} = \rho(z)\frac{\partial^2 u_z}{\partial t^2} \tag{5}$$

In order to obtain an exact analytical solution, variations in the material properties of the FGM thin film are assumed to be expressed as exponential functions of the space variable:

$$k(z) = k_0 e^{az},\ C(z) = C_0 e^{bz},\ \rho(z) = \rho_0 e^{qz},\ [\lambda(z), \mu(z)]=[\lambda_0, \mu_0]e^{pz},\ \alpha(z)=\alpha_0 e^{fz} \tag{6}$$

where k_0, C_0, ρ_0 λ_0, μ_0, and α_0 are material properties at the top surface of the thin film and a, b, q, p and f are nonhomogeneous parameters.

The governing equations (2) and (5) are solved by applying the following transformations:

$$\zeta = e^{\delta z},\ [T^*(z,s), u^*(z,s), \sigma^*(z,s)] = \int_0^\infty [T(z,t), u(z,t), \sigma(z,t)]e^{-st}dt \tag{7}$$

where δ is an arbitrary constant and s is the Laplace parameter.

Assuming that $b+q-a=2\delta$, $q-p=2\delta$, $f=2\delta$, and $\gamma=-a/2\delta=-p/2\delta$, the solutions are given by

$$T^* = A\zeta^\gamma I_{-\gamma}(\omega_1\zeta) + B\zeta^\gamma K_{-\gamma}(\omega_1\zeta) \tag{8}$$

$$u_z^* = \zeta^\gamma[CI_{-\gamma}(\omega_1\zeta) + DK_{-\gamma}(\omega_1\zeta) + \zeta\{FI_{-\gamma+1}(\omega_1\zeta) + GK_{-\gamma+1}(\omega_1\zeta)\} + LI_{-\gamma}(\omega_2\zeta) + MK_{-\gamma}(\omega_2\zeta)] \tag{9}$$

where $I_{-\gamma}(\zeta)$ and $K_{-\gamma}(\zeta)$ are modified Bessel's functions, A, B, L, and M are unknown coefficients to be determined from the boundary conditions, C, D, F, and G are coefficients related to A and B, $\omega_1 = \sqrt{C_0\rho_0 s}/\delta\sqrt{k_0}$, and $\omega_2 = s/\delta$.

The inversions of Eqs. (8) and (9) have been obtained for the cases of $\gamma = \pm 0.5$.

NUMERICAL RESULTS

Numerical calculations have been carried out for the following conditions.

$$l = 10^{-8}\,\mathrm{m},\ \rho_0 = 2700\,\mathrm{kg\,m^{-3}},\ k_0 = 155\,\mathrm{W\,m^{-1}K^{-1}},$$
$$C_0 = 964\,\mathrm{J\,kg^{-1}K^{-1}},\ \alpha_0 = 23.4\times10^{-6}\,\mathrm{K^{-1}},$$
$$(\lambda_0, \mu_0) = (50.3, 25.9)\times10^9\,\mathrm{N\,m^{-2}},\ \delta l = 0.4$$

The time histories of stresses are illustrated in Figs. 2 and 3, where the dimensionless quantities are

$$\bar{z} = \frac{z}{l},\ \bar{t} = \sqrt{\frac{\lambda_0 + 2\mu_0}{\rho_0}}\frac{t}{l},\ \bar{\sigma}_{zz} = \frac{\sigma_{zz}}{(3\lambda_0 + 2\mu_0)\alpha_0 T_c}$$

Figures 2 and 3 show that the stress oscillation is periodically monotonic for the case of $\gamma = 0.5$, while it is unsteady for the case of $\gamma = -0.5$. The analytical and numerical results reveal that the mechanical impedance $Z=\sqrt{\{\lambda(z)+2\mu(z)\}\rho(z)}$ is constant when $\gamma = 0.5$, but it is a function of the space variable when $\gamma = -0.5$.

The same problem has been analyzed numerically by applying the method of characteristics. The results of the numerical solutions are found to be in excellent agreement with those of the analytical solutions. For a realistic case, the stress oscillation is also obtained for an FGM thin film composed of Y-TPZ and MgO. These results are omitted here.

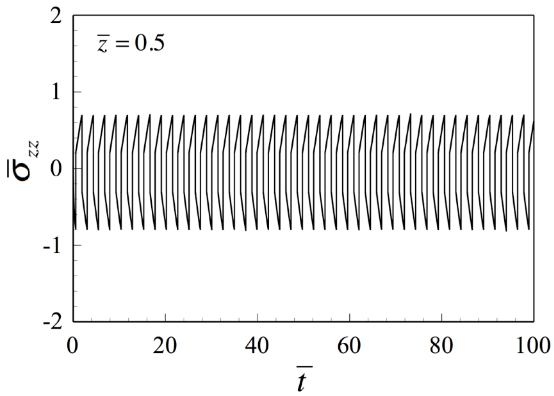

Fig. 2 Stress oscillation for case of $\gamma = 0.5$

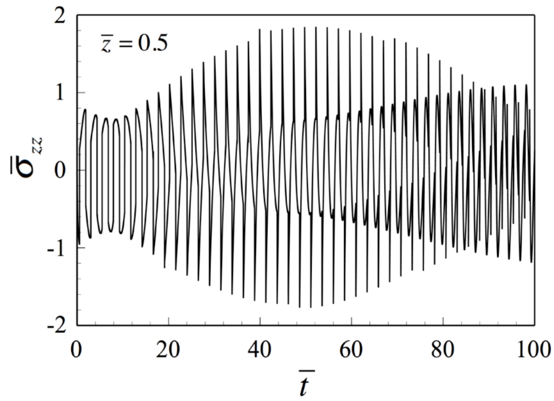

Fig. 3 Stress oscillation for case of $\gamma = -0.5$