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STATIC AND DYNAMIC BENDING OF RECTANGULAR SHEET OF BIOMATERIAL

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Soft biological tissues are increasingly being investigated in applied engineering science for their practical relevance in medical applications. In this context, this work explores the static and dynamic bending of a rectangular sheet made of a hyperelastic material which stands as a possible modelling of adventitia in human arteries. Both physical and geometrical nonlinearities are accounted for, respectively described through the Novozhilov nonlinear plate equations [1] and the Neo-Hookean, Mooney-Rivlin, and Ogden hyperelastic laws [2,3].

A so-called local models method (LMM) is purposely developed for the problem of interest [4]. This method consists in expanding the non-polynomial strain energy densities found in hyperelastic laws into a truncated power series expressed in the strain components. This yields an approximate polynomial expression which provides direct access to all common techniques targeting physically linear materials [1]. However, due to the truncation, such derivations are only able to capture the plate dynamical behavior locally, that is in the vicinity of a deformed static configuration. In order to reach a highly distorted state, several successive local models with cubic nonlinearities are built. The LMM solutions are verified through a systematic comparison with the exact static and dynamic solutions for simple cases of the problem with small number degrees of freedom and good agreement is observed.

The Neo-Hookean and Mooney-Rivlin laws properly capture the behavior of the actual material up to moderate strains only. However, the Ogden law correctly reproduces real biomaterial behavior at high strains (up to 45%), including the known sharp increase in stiffness [5] (Fig. 1).

For the sheet static deflection it is found that at small strains all hyperelastic laws produce results similar to physically linear material. The Neo-Hookean and Mooney-Rivlin laws produce very similar pressure-deflection curves. The curve predicted by the Ogden model deviates from them at high strains.

Also, the nonlinear vibrations of the plate are explored around a highly pre-loaded static state where the effect of physical nonlinearity cannot be neglected. The initial deformed configuration involves a principal bending coordinate equal to 70 thicknesses and a local model constructed around this configuration is used to study the dynamics. The nonlinearity of the system is found to be very weak (Fig. 2). We can conclude that the only law that is able to reproduce the real biomaterial behavior is the Ogden's law. Despite of the significant difference in static deflection, all three models sheet exhibit very weak dynamic nonlinearity. Based on the results of this study the dynamics of artery, modeled as multilayered circular cylindrical shell, will be carried out.

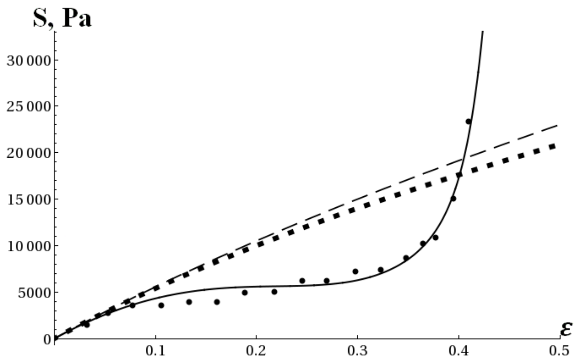

Figure 1. Stress versus Green-Lagrange strain for uniaxial tension. Dotted line, Neo-Hookean law; dashed line, Mooney-Rivlin law; continuous line, Ogden law; ●, experimental point.

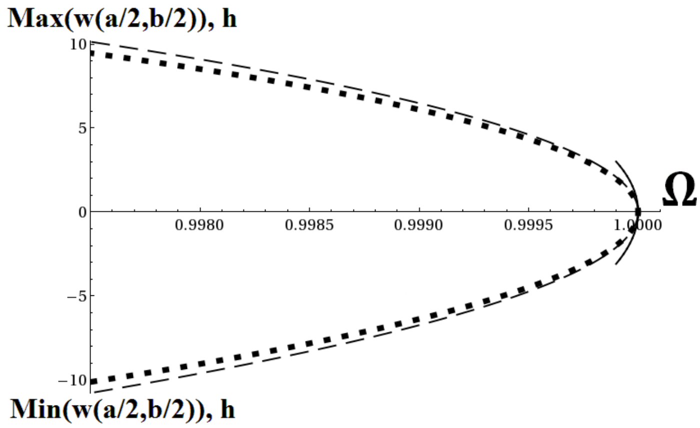

Figure 2. Backbone curves for free vibrations around the deformed configuration for the biomaterial described by different hyperelastic laws. Non-dimensional frequencies, normalized with respect to the first natural frequency of the corresponding deformed plate, are shown in abscissa. Dotted line, Neo-Hookean model; dashed line, Mooney-Rivlin model; solid line, Ogden model.

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