Further exploration of the Lagrangian CSTS (Conservative Space- and Time-Staggered) hydrodynamic scheme

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In the context of Lagrangian computation of compressible fluid dynamics, space- and time-staggered (STS) schemes have been widely used since the pioneering work of von Neumann and Richtmyer [5].

These schemes are known to be non conservative in total energy and present some distortions for accurately capturing Hugoniot relationships at shock discontinuities (on shock level and on propagation velocity). Corrections have thus been brought to the simplest STS scheme [3,4] in order to recover energy conservation. Such corrections appear to have been seldom exploited because of possible detrimental effects [3]: loss of the explicit character of the momentum equation, non positive quadratic form of the kinetic energy, etc.

This has also motivated the development of time-centered space-staggered, or even time- and space-centered schemes in order to have total energy conservation and despite sometimes significant complications of the algorithms.

In this work, in the spirit of Burton [4], we propose a modification of the basic STS scheme, denoted by CSTS scheme, which is conservative, entropic, compatible, positive definite in kinetic energy, explicit in the momentum equation, and second-order accurate (even for variable time steps).

The ability of the CSTS scheme to correct the energy non conservation defect of the STS scheme has been demonstrated [1,2]. In particular, the good behaviour we have observed on numerical tests for larger time steps than the usual STS scheme suggests better stability properties that we shall explore. Moreover, following the spirit for the construction of the scheme, we will get the extension to the axisymmetric geometry and we will discuss the question of preserving 1D symmetry for cylindrical spherical flows.

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