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BLOCK STRATEGIES TO SPEED UP CONVERGENCE IN NON-OVERLAPPING DOMAIN DECOMPOSITION METHODS

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In the context of massively parallel simulations for structure mechanics, besides floating-point operations, a significant amount of time is spent in communications between processors. Communication time can be decomposed in latency time which in the end is proportional to the number of exchanges, and data transfer in itself which is proportional to the quantity of data exchanged (and inversely proportional to the bandwidth). Avoiding communication, even without lowering the number of floating point operations or the amount of data exchanged, is then a way to design efficient solution strategies. For instance, in [3], communication-avoiding Krylov solvers are derived by exploiting the sparsity of the matrices.

The context of our study is non-overlapping domain decomposition methods, like FETI or BDD [4, 5], where a condensed interface system is formulated and solved by a preconditioned Krylov iterative solver [6], and where communications occur when doing matrix-vector multiplications and vector dot products. We take advantage of the additive structure of the operator and of the preconditioner in order to derive two block strategies where each iteration involves more floating point operations and bigger amount of data to exchange but the same number of exchanges, the objective being to drastically decrease the number of iterations. The idea of building a block-like FETI iteration was already proposed in [7] where it was shown that in case of 2 subdomains the number of operation for the block and the standard FETI were similar.

The first method exploits the additivity of the problem in order to derive a set of right-hand-sides solved with classical block solver (brhs). The second techniques exploits the additivity of the preconditioner to define a block of search directions (bsd); it generalizes what was proposed in [2] for structure made out of repeated patterns, and it is connected to [1] which was proposed in the framework of restricted additive Schwarz using GMRes.

The presentation will present the block algorithms more in details, will give more results and will discuss the numerical difficulties that must be tackled when implementing the methods.

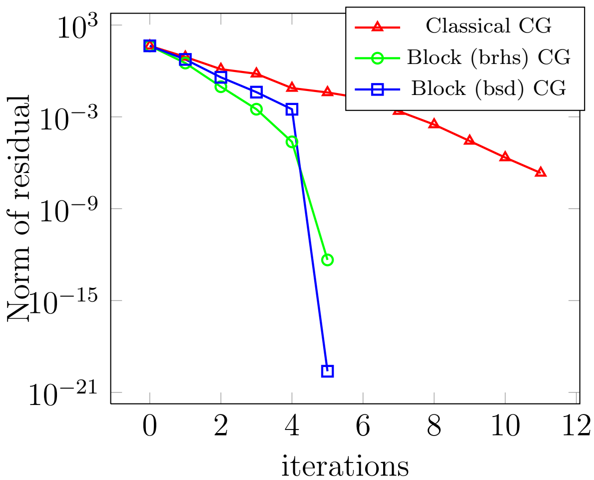

Figure 1: Block strategies compared to classical CG on an academical test.

Figure 1 presents first results for a very academical test (2D linear mechanics, 3×3 square subdomains, 100 interface dofs, volume loading) conducted on a `octave` code. We see that classical CG has a regular convergence whereas the block strategies experience a fast decrease of the residual after 5 iterations.

REFERENCES

[1] C. Greif, T. Rees, D. B. Szyld. Additive Schwarz with Variable Weights, Technical Report 12-11-30, Department of Mathematics, Temple University, 2012.

[2] P. Gosselet, D. J. Rixen, C. Rey. A domain decomposition strategy to efficiently solve structures containing repeated patterns. *International Journal for Numerical Methods in Engineering* **78**, 828–842, 2009.

[3] E. Carson, N. Knight, J. Demmel.Avoiding communication in nonsymmetric Lanczos-based Krylov subspace methods. *SIAM J. Sci. Comput.* **35**, S42S61, 2013.

[4] C. Farhat, F.-X. Roux. A method of finite element tearing and interconnecting and its parallel solution algorithm. *International Journal for Numerical Methods in Engineering* **32** 1991.

[5] J. Mandel. Balancing domain decomposition. *Communications in Numerical Methods in Engineering* **9**, 1993.

[6] Y. Saad. *Iterative methods for sparse linear systems.* PWS Publishing Company, 3rd edition, 2000.

[7] D. Rixen. *Substructuring and Dual Methods in Structural Analysis.* PhD thesis of Université de Liège, Belgium. Collection des Publications de la Faculté des Sciences appliquées, n° 175, 1997.