11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

ENERGY-STABLE GALERKIN REDUCED ORDER MODELS FOR NONLINEAR COMPRESSIBLE FLOW

Irina Kalashnikova[1], Matthew Barone[2], Srinivasan Arunajatesan[2], Bart van Bloemen Waanders[3] and Jeffrey Fike[4]

[1] Computational Mathematics Department, Sandia National Laboratories, P.O. Box 5800, MS 1320, Albuquerque, NM, U.S.A. (ikalash@sandia.gov).
[2] Aerosciences Department, Sandia National Laboratories, P.O. Box 5800, MS 0825, Albuquerque, NM, U.S.A.
[3] Optimization and Uncertainty Quantification Department, Sandia National Laboratories, P.O. Box 5800, MS 1318, Albuquerque, NM, U.S.A.
[4] Mechanical Engineering Department, Sandia National Laboratories, P.O. Box 5800, MS 1070, Albuquerque, NM, U.S.A.

Key words: *Reduced order model (ROM), proper orthogonal decomposition (POD), Galerkin projection, energy-stability, nonlinear compressible flow.*

Computational fluid dynamics (CFD) simulations of the fidelity required in modern-day applications can possess tens of millions of unknowns. Even with improved algorithms and the availability of massively-parallel supercomputers, these high-fidelity models are often too computationally expensive for use in a design or analysis setting: it can take on the order of weeks to obtain the result of a long-time transient simulation [2]. Reduced order models (ROMs) are derived from the dynamics of high-fidelity CFD simulations, but possess far fewer degrees of freedom. Since ROMs are, by construction, small and fast, model reduction is a promising tool that can enable on-the-spot decision-making, optimization, control and/or uncertainty quantification.

A popular approach to building ROMs for fluid problems is the proper orthogonal decomposition (POD)/Galerkin projection method. This method consists of two steps: (1) the computation of the POD basis from a set of snapshots of the solution field, followed by (2) the Galerkin projection of the governing equations onto this reduced basis in some inner product. POD is a mathematical procedure that constructs a reduced basis for an ensemble of snapshots collected from a high-fidelity simulation [4]. This basis is optimal in the sense that it describes more energy on average of the ensemble than any other linear basis of the same reduced dimension. Unfortunately, ROMs constructed via the POD/Galerkin method lack, in general, an *a priori* stability guarantee. This leads to practical limitations of ROMs obtained using the POD/Galerkin method: a ROM aimed to capture a flow in a physically stable (i.e., bounded as $t \to \infty$) regime might be stable for a given number of modes, but unstable (i.e., unbounded as $t \to \infty$), and therefore inaccurate, for other choices of basis size [1, 2].

The present work focuses on approaches for building energy-stable POD/Galerkin ROMs for compressible flow problems, which are often plagued with instabilities. In [1, 2], it was shown that the inner product used to define the Galerkin projection step of the model reduction procedure is closely tied to the stability of the resulting ROM. This inner product (termed the "energy inner product") was derived for the linearized compressible Euler and Navier-Stokes equations. The primary contribution of the present work is the derivation of a stability-preserving energy inner product for the full *nonlinear* compressible Navier-Stokes equations. It is demonstrated that if the Galerkin projection step of the model reduction is performed in this inner product, whose associated norm represents the total energy of the fluid system, the ROM numerical solution will be energy-stable, that is, bounded in a way that is consistent with the behavior of the energy of the exact solution to the governing fluid equations. This stability result holds not only for the POD basis, but for *any* choice of reduced basis. The performance of the proposed model reduction approach is evaluated on some test cases involving viscous flow over an open cavity, and compared to other energy-based model reduction approaches, e.g., ROMs constructed for the isentropic compressible Navier-Stokes equations in an energy inner product proposed by Rowley *et al.* [3]. The effects of boundary conditions on ROM stability are also examined in the context of these test cases.

REFERENCES

[1] M.F. Barone, I. Kalashnikova, D.J. Segalman, H. Thornquist. Stable Galerkin Reduced Order Models for Linearized Compressible Flow. *J. Comput. Phys.* **288** 1932–1946, 2009.

[2] I. Kalashnikova, S. Arunajatesan. A Stable Galerkin Reduced Order Model (ROM) for Compressible Flow, *WCCM-2012-18407, 10th World Congress on Computational Mechanics (WCCM)*, Sao Paulo, Brazil, 2012.

[3] C.W. Rowley, T. Colonius, R.M. Murray. Model reduction for compressible flows using POD and Galerkin projection. *Physica D. Nonlinear Phenomena* **189** (12) 115–129, 2004.

[4] J.L. Lumley. Stochastic tools in turbulence. Academic Press: New York, 1971.