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LOCAL ESHELBY MATRIX ENHANCED EIGENSTRAIN BIE FOR 3-D ANALYSIS OF PARTICLES IN FULL SPACE

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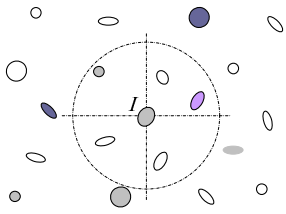
Determination of elastic behaviour of inclusions embedded in a matrix is of considerable significance in a wide variety of physical and engineering problems [1-2]. Following the pioneering work of Eshelby [3], inclusion problems have been a focus of solid mechanics for several decades. For multiple particle problems, however, there would be still great challenges encountered for both the analytical [4] and numerical [5] methods. Recently a new computational model with an iteration procedure has been proposed by introducing the concepts of Eshelby's eigenstrain and the equivalent inclusion into the boundary integral equations (BIE) for the 2-D and 3-D stress analysis and the overall properties of solids with a large number of particles [6-7]. For the densely populated particles, however, the interaction among particles will affect the convergence of iteration, depending on the distance between particles. Shorter the distance is, stronger the interaction will be. In order to overcome this difficulty, the local Eshelby matrix for the newly defined group of near field particles has been introduced with the aid of the discrete form of eigenstrain BIE in full space to improve the original algorithm. Taking the sub-domain BEM in full space [8] as the control, the 3-D stress analysis has been carried out for several ellipsoidal particles with various Young's modulus ratio (E_I/E_M) and different shapes to verify the feasibility and efficiency of the improved eigenstrain BIE algorithm.



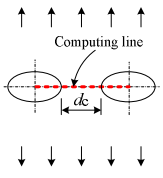


Fig.1 Group definitions for multiple particles

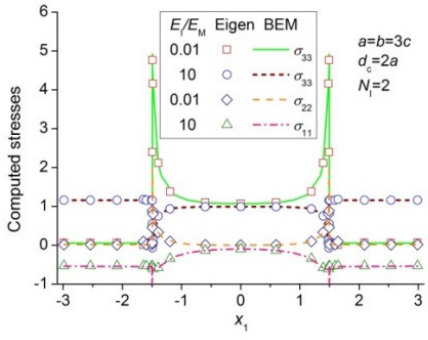


Fig.2 The stresses of two oblate particles in full space

As shown in Fig. 1, the particles with a number N_L within the circle are defined as the group of near-field particles for the current particle I while others outside the circle are that of the far-field particles. By combining the constitutive relation and considering only the effect of near-field particles while tentatively neglecting the effect of far-field particles, the stress BIE

in full space can be written as

$$\sigma_{ij}^{I} = E_{ijkl}\varepsilon_{kl}^{I} = E_{ijkl}\left(\varepsilon_{kl}^{CI} - \varepsilon_{kl}^{0I}\right) = \sum_{J=1}^{N_L}\int_{\Gamma_J} \varepsilon_{kl}^{0J} x_l \tau_{ijk}^{*} d\Gamma \tag{1}$$

where ε^{0I}_{ij}, ε^{CI}_{ij}, and ε^{I}_{ij} are respectively the eigen-, the constrained and the applied strains. E_{ijkl} stands for the elastic tensor of the matrix. It is noticed in full space that the boundary integrals vanish in Eqn. (1). The constrained strain of the current particle and the replacement of equivalent inclusion can respectively be expressed as

$$\varepsilon_{ij}^{CI} = \varepsilon_{ij}^{0I} + \sum_{J=1}^{N_L} C_{ijkl}^{J}\int_{\Gamma_J} \varepsilon_{mn}^{0J} x_n \tau_{klm}^{*} d\Gamma = \sum_{J=1}^{N_L} S_{ijkl}^{IJ}\varepsilon_{kl}^{0J} \tag{2}$$

$$(1-\beta_1)\varepsilon_{ij}^{CI} + \beta_2\delta_{ij}\varepsilon_{kk}^{CI} - \varepsilon_{ij}^{0I} - \frac{\nu_M}{1-2\nu_M}\delta_{ij}\varepsilon_{kk}^{0I} = -(1-\beta_1)\varepsilon_{ij}^{I} - \beta_2\delta_{ij}\varepsilon_{kk}^{I}, \quad I = 1,2,\cdots,N_L \tag{3}$$

where S^{IJ}_{ijkl} and C^{J}_{ijkl} are the Eshelby and the compliance tensors, β_1 and β_2 the material constants, respectively. The so-called Eshelby matrix is the discrete form of Eqn. (3) with (2) after dividing boundary and interfaces using quadratic elements. In general, every particle has a unique Eshelby matrix correspondingly once the radius of circle being given (Fig. 1). In this way, the stress and the applied strain responses of a particle consist of the effects from three parts: the applied load, the near- and the far-field groups.

Table 1 The mean CPU time(s) of two algorithms

N_I	2	3	5
Sub-domain BEM	190	422	1265
Eigenstrain	1.69	2.38	6.27
The CPU ratio	112	177	202

As one of the examples, the stress distributions of two oblate particles as well as the CPU times are compared in Fig. 2 and Table 1, respectively, where N_I is the total number of particles in computation, showing the accuracy, feasibility and high efficiency of the proposed eigenstrain BIE algorithm.

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